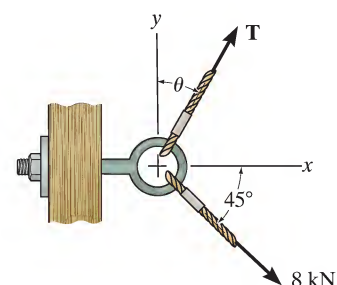


# Vectores fuerza

2

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•2–1. If  $\theta = 30^\circ$  and  $T = 6 \text{ kN}$ , determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive  $x$  axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{6^2 + 8^2 - 2(6)(8)\cos 75^\circ} \\ = 8.669 \text{ kN} = 8.67 \text{ kN}$$

Ans.

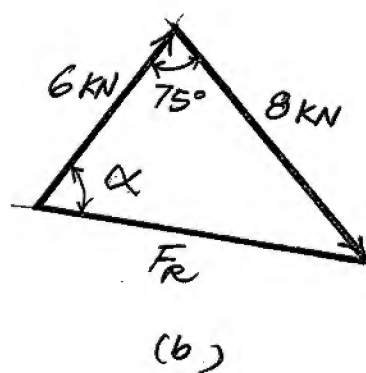
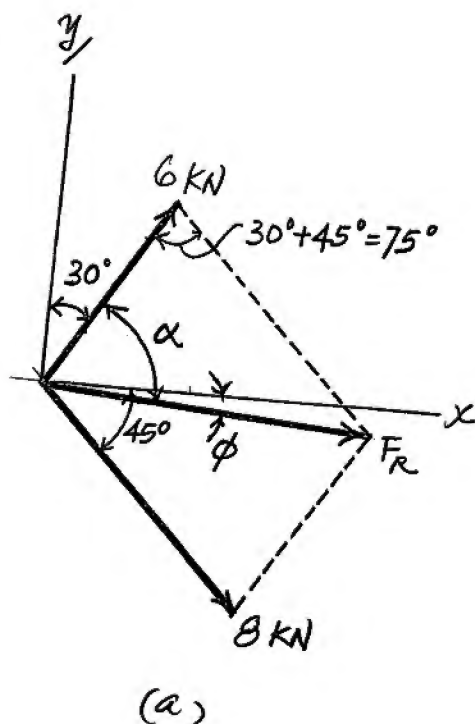
Applying the law of sines to Fig. *b* and using this result, yields

$$\frac{\sin \alpha}{8} = \frac{\sin 75^\circ}{8.669} \quad \alpha = 63.05^\circ$$

Thus, the direction angle  $\phi$  of  $\mathbf{F}_R$  measured clockwise from the positive  $x$  axis is

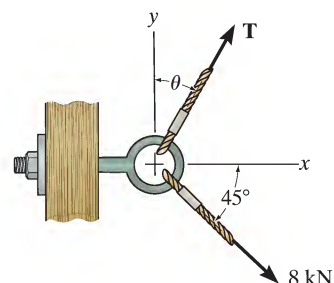
$$\phi = \alpha - 60^\circ = 63.05^\circ - 60^\circ = 3.05^\circ$$

Ans.



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2-2. If  $\theta = 60^\circ$  and  $T = 5 \text{ kN}$ , determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive  $x$  axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{5^2 + 8^2 - 2(5)(8)\cos 105^\circ}$$

$$= 10.47 \text{ kN} = 10.5 \text{ kN}$$

Ans.

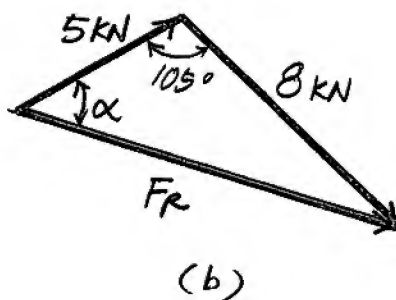
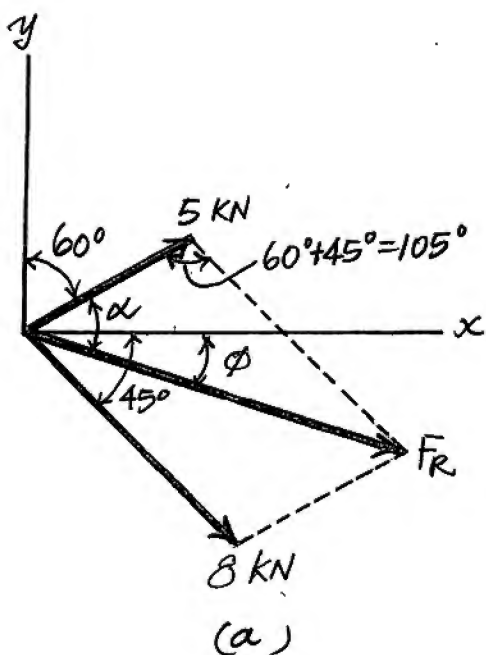
Applying the law of sines to Fig. *b* and using this result, yields

$$\frac{\sin \alpha}{8} = \frac{\sin 105^\circ}{10.47} \quad \alpha = 47.54^\circ$$

Thus, the direction angle  $\phi$  of  $F_R$  measured clockwise from the positive  $x$  axis is

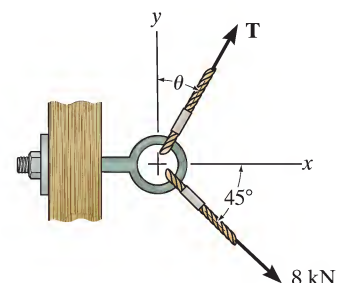
$$\phi = \alpha - 30^\circ = 47.54^\circ - 30^\circ = 17.5^\circ$$

Ans.



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2-3. If the magnitude of the resultant force is to be 9 kN directed along the positive  $x$  axis, determine the magnitude of force  $T$  acting on the eyebolt and its angle  $\theta$ .



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$T = \sqrt{8^2 + 9^2 - 2(8)(9)\cos 45^\circ} \\ = 6.571 \text{ kN} = 6.57 \text{ kN}$$

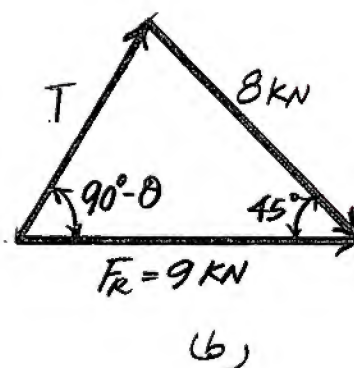
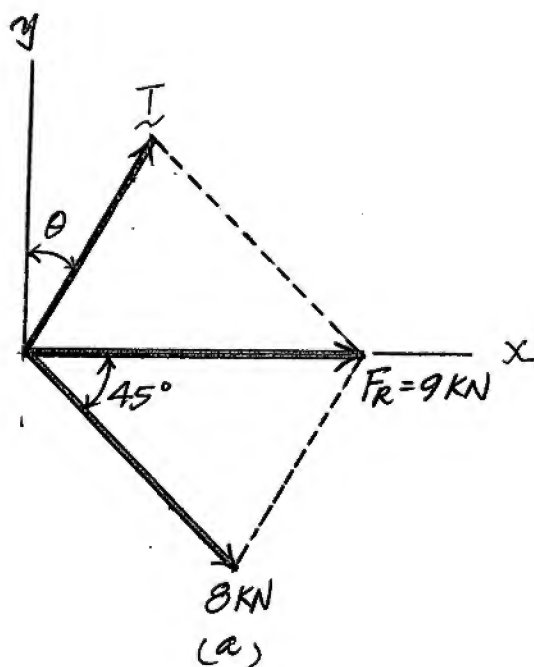
Ans.

Applying the law of sines to Fig. *b* and using this result, yields

$$\frac{\sin(90^\circ - \theta)}{8} = \frac{\sin 45^\circ}{6.571}$$

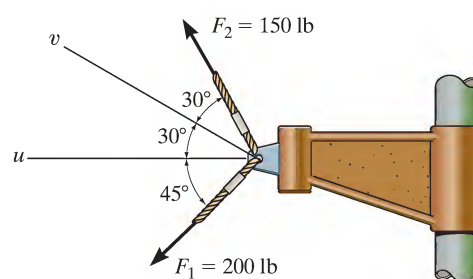
$$\theta = 30.6^\circ$$

Ans.



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\*2-4. Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive  $u$  axis.



The parallelogram law of addition and the triangular rule are shown in Figs.  $a$  and  $b$ , respectively.

Applying the law of cosines to Fig.  $b$ ,

$$F_R = \sqrt{200^2 + 150^2 - 2(200)(150)\cos 75^\circ}$$

$$= 216.72 \text{ lb} = 217 \text{ lb}$$

Ans.

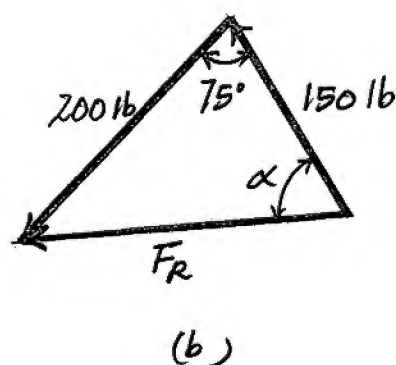
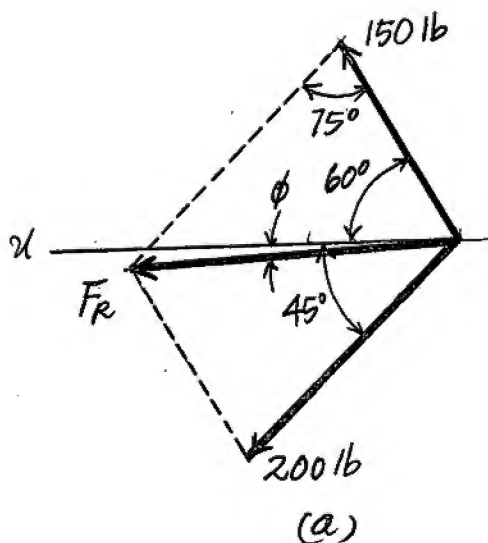
Applying the law of sines to Fig.  $b$  and using this result yields

$$\frac{\sin \alpha}{200} = \frac{\sin 75^\circ}{216.72} \quad \alpha = 63.05^\circ$$

Thus, the direction angle  $\phi$  of  $F_R$ , measured counterclockwise from the positive  $u$  axis, is

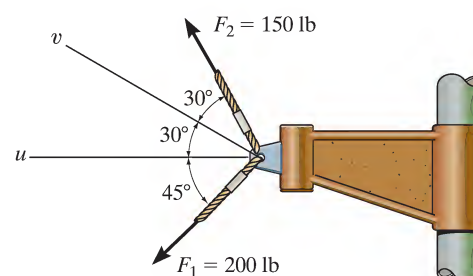
$$\phi = \alpha - 60^\circ = 63.05^\circ - 60^\circ = 3.05^\circ$$

Ans.



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- 2–5. Resolve  $\mathbf{F}_1$  into components along the  $u$  and  $v$  axes, and determine the magnitudes of these components.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

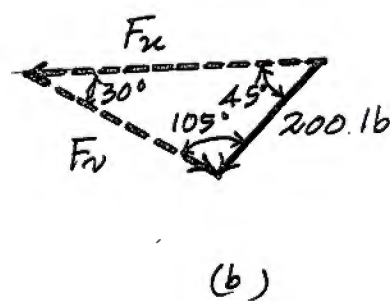
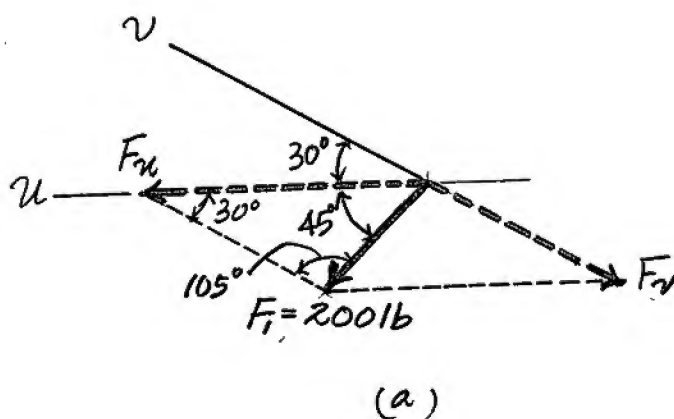
Applying the law of sines to Fig. *b*, yields

$$\frac{F_u}{\sin 105^\circ} = \frac{200}{\sin 30^\circ} \quad F_u = 386 \text{ lb}$$

Ans.

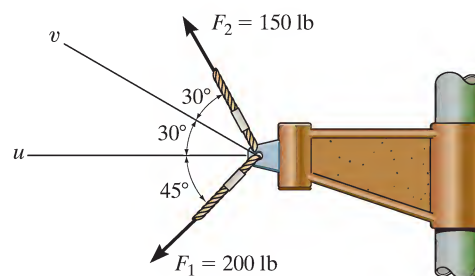
$$\frac{F_v}{\sin 45^\circ} = \frac{200}{\sin 30^\circ} \quad F_v = 283 \text{ lb}$$

Ans.



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2-6. Resolve  $F_2$  into components along the  $u$  and  $v$  axes, and determine the magnitudes of these components.

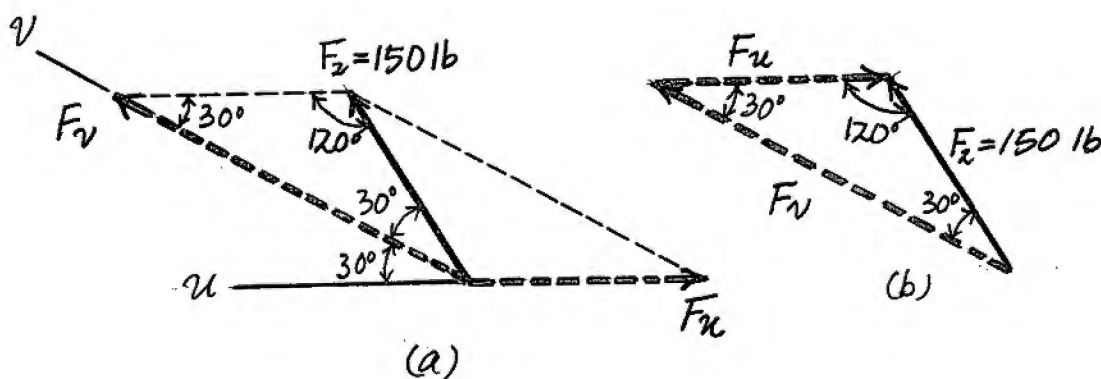


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of sines to Fig. *b*,

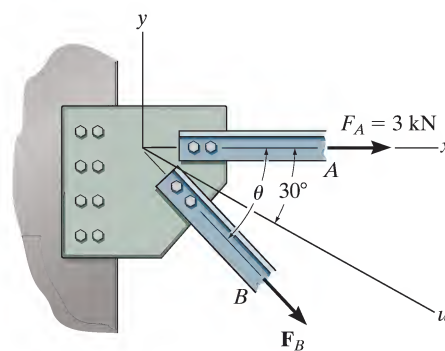
$$\frac{F_u}{\sin 30^\circ} = \frac{150}{\sin 30^\circ} \quad F_u = 150 \text{ lb} \quad \text{Ans.}$$

$$\frac{F_v}{\sin 120^\circ} = \frac{150}{\sin 30^\circ} \quad F_v = 260 \text{ lb} \quad \text{Ans.}$$



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2-7. If  $F_B = 2$  kN and the resultant force acts along the positive  $u$  axis, determine the magnitude of the resultant force and the angle  $\theta$ .



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of sines to Fig. *b*, yields

$$\frac{\sin \phi}{3} = \frac{\sin 30^\circ}{2} \quad \phi = 48.59^\circ$$

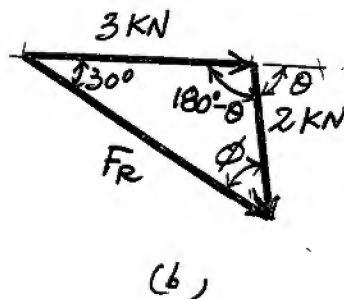
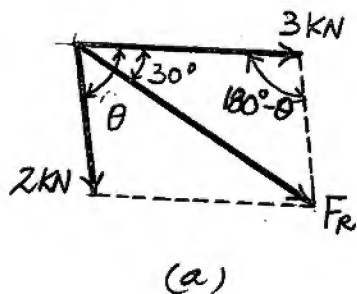
Thus,

$$\theta = 30^\circ + \phi = 30^\circ + 48.59^\circ = 78.59^\circ = 78.6^\circ$$

Ans.

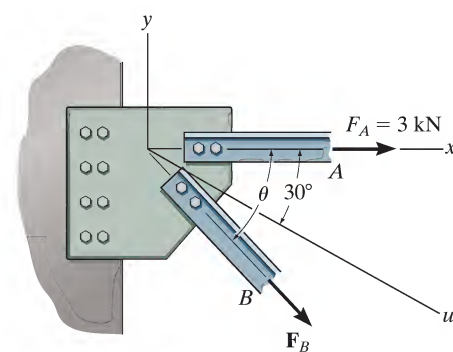
With the result  $\theta = 78.59^\circ$ , applying the law of sines to Fig. *b* again, yields

$$\frac{F_R}{\sin(180^\circ - 78.59^\circ)} = \frac{2}{\sin 30^\circ} \quad F_R = 3.92 \text{ kN} \quad \text{Ans.}$$



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**\*2–8.** If the resultant force is required to act along the positive  $u$  axis and have a magnitude of 5 kN, determine the required magnitude of  $F_B$  and its direction  $\theta$ .



The parallelogram law of addition and the triangular rule are shown in Figs.  $a$  and  $b$ , respectively.

Applying the law of cosines to Fig.  $b$ ,

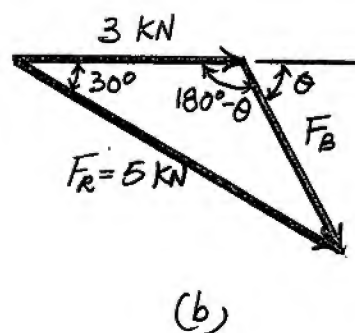
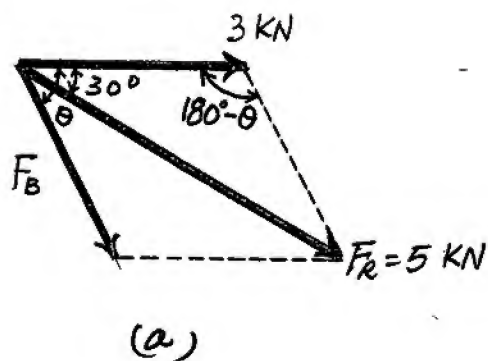
$$F_B = \sqrt{3^2 + 5^2 - 2(3)(5)\cos 30^\circ}$$

$$= 2.832 \text{ kN} = 2.83 \text{ kN}$$

**Ans.**

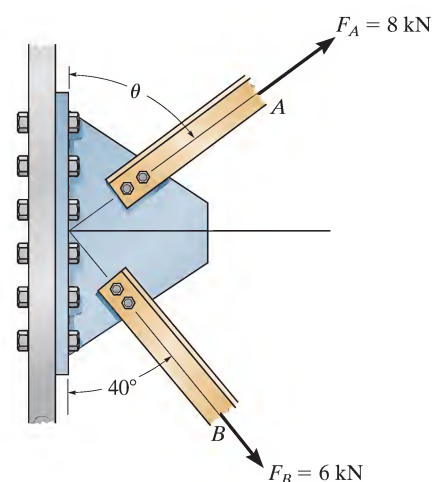
Using this result and realizing that  $\sin(180^\circ - \theta) = \sin\theta$ , the application of the sine law to Fig.  $b$ , yields

$$\frac{\sin\theta}{5} = \frac{\sin 30^\circ}{2.832} \quad \theta = 62.0^\circ \quad \text{Ans.}$$



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•2–9. The plate is subjected to the two forces at  $A$  and  $B$  as shown. If  $\theta = 60^\circ$ , determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



**Parallelogram Law** : The parallelogram law of addition is shown in Fig. (a).

**Trigonometry** : Using law of cosines [Fig. (b)], we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle  $\theta$  can be determined using law of sines [Fig. (b)].

$$\frac{\sin \theta}{6} = \frac{\sin 100^\circ}{10.80}$$

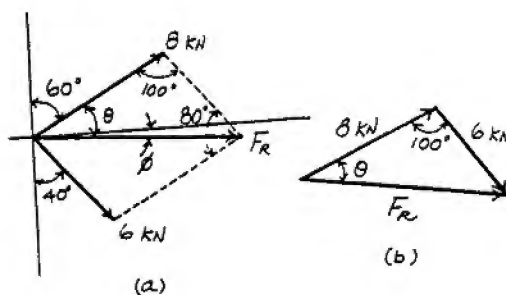
$$\sin \theta = 0.5470$$

$$\theta = 33.16^\circ$$

Thus, the direction  $\phi$  of  $F_R$  measured from the  $x$  axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ$$

Ans

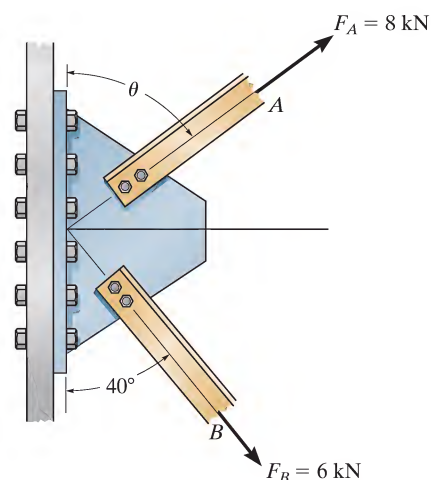


(a)

(b)

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**2–10.** Determine the angle of  $\theta$  for connecting member  $A$  to the plate so that the resultant force of  $\mathbf{F}_A$  and  $\mathbf{F}_B$  is directed horizontally to the right. Also, what is the magnitude of the resultant force?



**Parallelogram Law :** The parallelogram law of addition is shown in Fig. (a).

**Trigonometry :** Using law of sines [Fig. (b)], we have

$$\frac{\sin (90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

$$\sin (90^\circ - \theta) = 0.5745$$

$$\theta = 54.93^\circ = 54.9^\circ$$

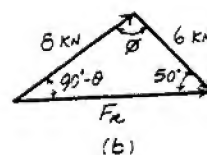
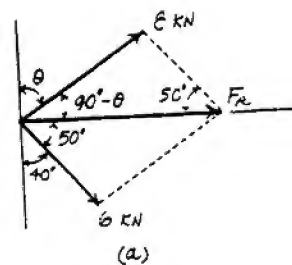
**Ans**

From the triangle,  $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$ . Thus, using law of cosines, the magnitude of  $\mathbf{F}_R$  is

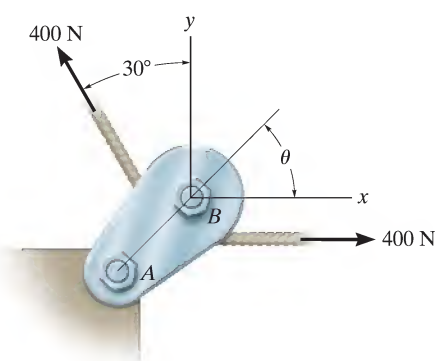
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6) \cos 94.93^\circ}$$

$$= 10.4 \text{ kN}$$

**Ans**

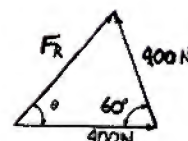
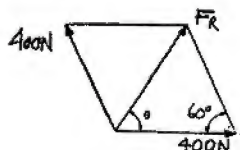


**2–11.** If the tension in the cable is 400 N, determine the magnitude and direction of the resultant force acting on the pulley. This angle is the same angle  $\theta$  of line  $AB$  on the tailboard block.



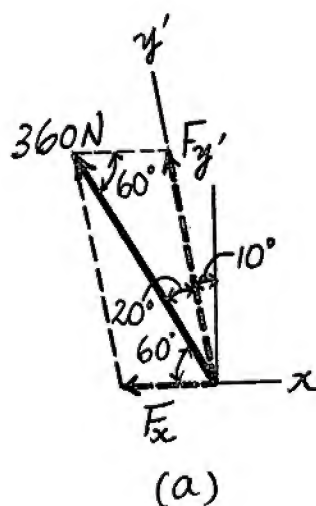
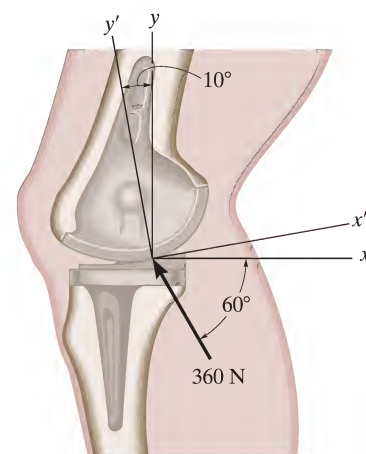
$$F_R = \sqrt{(400)^2 + (400)^2 - 2(400)(400) \cos 60^\circ} = 400 \text{ N} \quad \text{Ans}$$

$$\frac{\sin \theta}{400} = \frac{\sin 60^\circ}{400}; \quad \theta = 60^\circ \quad \text{Ans}$$



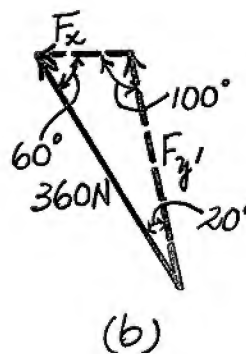
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**\*2-12.** The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the  $x$  and  $y'$  axes.



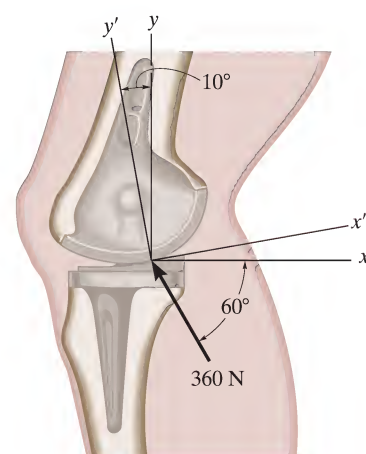
$$\frac{-F_x}{\sin 20^\circ} = \frac{360}{\sin 100^\circ}; \quad F_x = -125 \text{ N} \quad \text{Ans}$$

$$\frac{F_{y'}}{\sin 60^\circ} = \frac{360}{\sin 100^\circ}; \quad F_{y'} = 317 \text{ N} \quad \text{Ans}$$



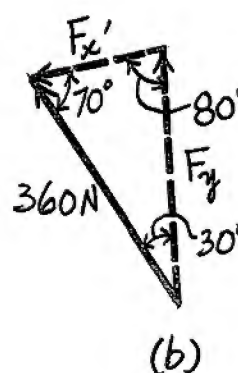
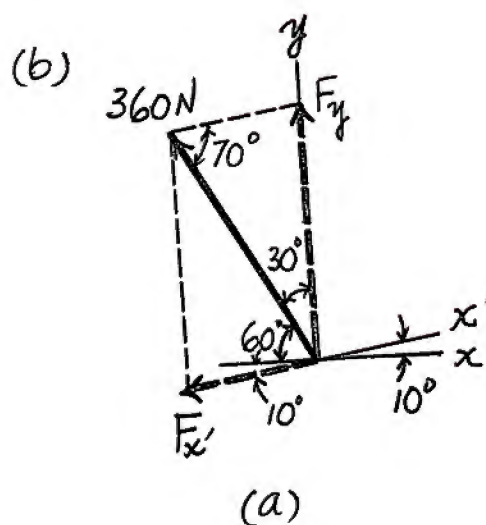
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- 2–13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the  $x'$  and  $y$  axes.



$$\frac{-F_{x'}}{\sin 30^\circ} = \frac{360}{\sin 80^\circ}; \quad F_{x'} = -183 \text{ N} \quad \text{Ans}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{360}{\sin 80^\circ}; \quad F_y = 344 \text{ N} \quad \text{Ans}$$



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2-14. Determine the design angle  $\theta$  ( $0^\circ \leq \theta \leq 90^\circ$ ) for strut  $AB$  so that the 400-lb horizontal force has a component of 500 lb directed from  $A$  towards  $C$ . What is the component of force acting along member  $AB$ ? Take  $\phi = 40^\circ$ .

**Parallelogram Law** : The parallelogram law of addition is shown in Fig. (a).

**Trigonometry** : Using law of sines [Fig. (b)], we have

$$\frac{\sin \theta}{500} = \frac{\sin 40^\circ}{400}$$

$$\sin \theta = 0.8035$$

$$\theta = 53.46^\circ = 53.5^\circ$$

Ans

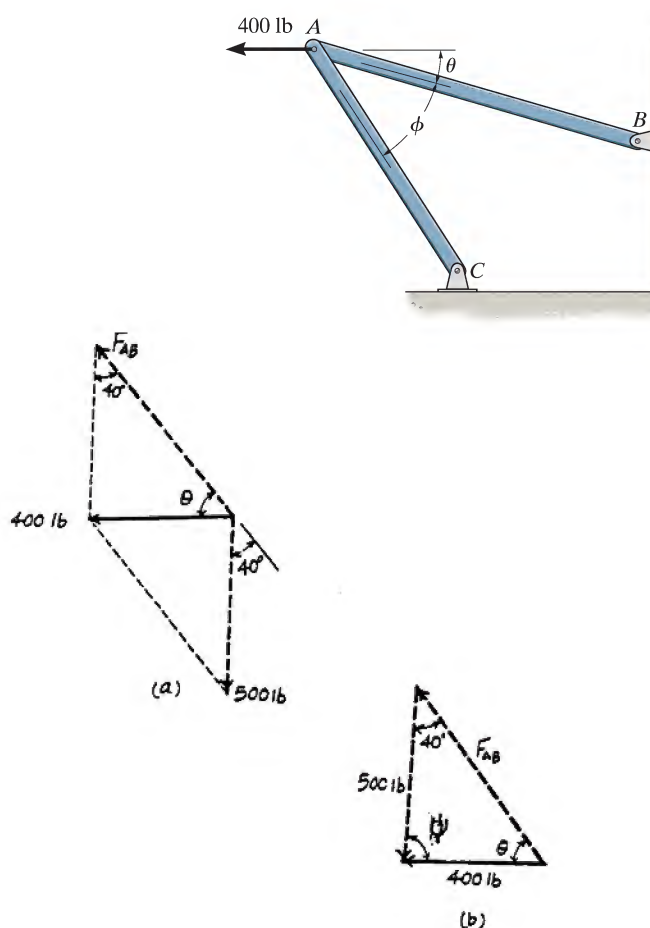
Thus,  $\psi = 180^\circ - 40^\circ - 53.46^\circ = 86.54^\circ$

Using law of sines [Fig. (b)]

$$\frac{F_{AB}}{\sin 86.54^\circ} = \frac{400}{\sin 40^\circ}$$

$$F_{AB} = 621 \text{ lb}$$

Ans



2-15. Determine the design angle  $\phi$  ( $0^\circ \leq \phi \leq 90^\circ$ ) between struts  $AB$  and  $AC$  so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from  $B$  towards  $A$ . Take  $\theta = 30^\circ$ .

**Parallelogram Law** : The parallelogram law of addition is shown in Fig. (a).

**Trigonometry** : Using law of cosines [Fig. (b)], we have

$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30^\circ} = 322.97 \text{ lb}$$

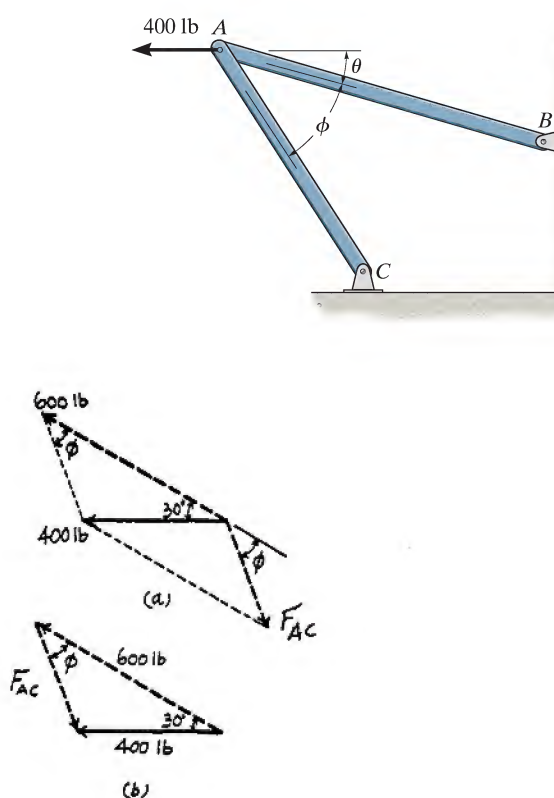
The angle  $\phi$  can be determined using law of sines [Fig. (b)].

$$\frac{\sin \phi}{400} = \frac{\sin 30^\circ}{322.97}$$

$$\sin \phi = 0.6193$$

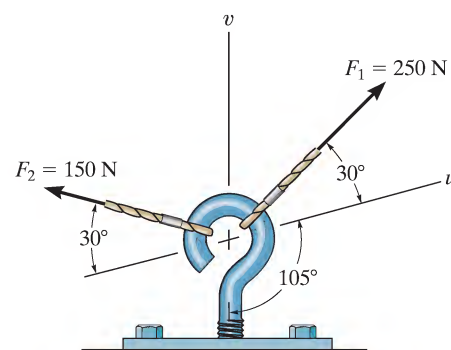
$$\phi = 38.3^\circ$$

Ans



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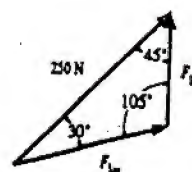
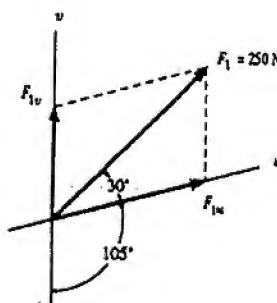
**\*2-16.** Resolve  $\mathbf{F}_1$  into components along the  $u$  and  $v$  axes and determine the magnitudes of these components.



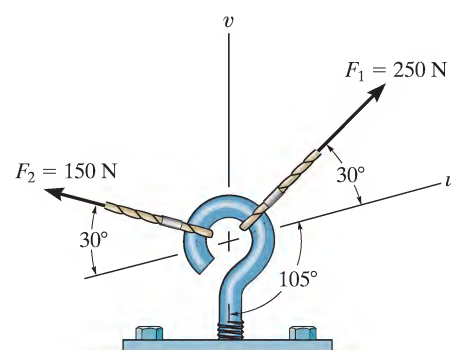
Sine law :

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{250}{\sin 105^\circ} \quad F_{1v} = 129 \text{ N} \quad \text{Ans}$$

$$\frac{F_{1u}}{\sin 45^\circ} = \frac{250}{\sin 105^\circ} \quad F_{1u} = 183 \text{ N} \quad \text{Ans}$$



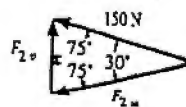
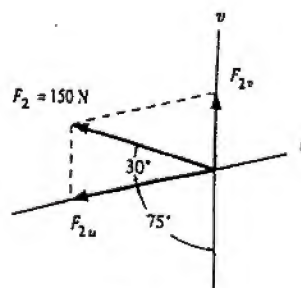
**•2-17.** Resolve  $\mathbf{F}_2$  into components along the  $u$  and  $v$  axes and determine the magnitudes of these components.



Sine law :

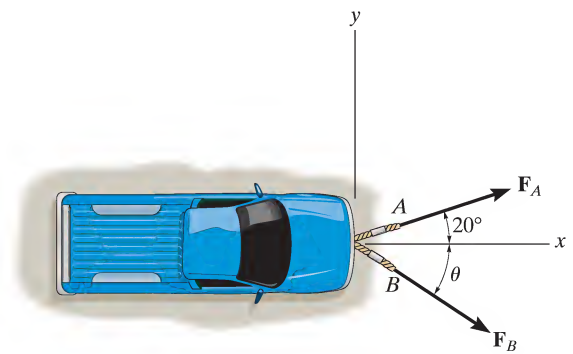
$$\frac{F_{2v}}{\sin 30^\circ} = \frac{150}{\sin 75^\circ} \quad F_{2v} = 77.6 \text{ N} \quad \text{Ans}$$

$$\frac{F_{2u}}{\sin 75^\circ} = \frac{150}{\sin 75^\circ} \quad F_{2u} = 150 \text{ N} \quad \text{Ans}$$



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**2–18.** The truck is to be towed using two ropes. Determine the magnitudes of forces  $F_A$  and  $F_B$  acting on each rope in order to develop a resultant force of 950 N directed along the positive  $x$  axis. Set  $\theta = 50^\circ$ .



**Parallelogram Law :** The parallelogram law of addition is shown in Fig. (a).

**Trigonometry :** Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 50^\circ} = \frac{950}{\sin 110^\circ}$$

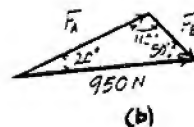
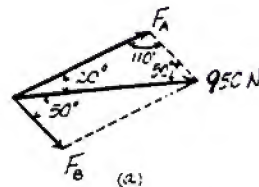
$$F_A = 774 \text{ N}$$

Ans

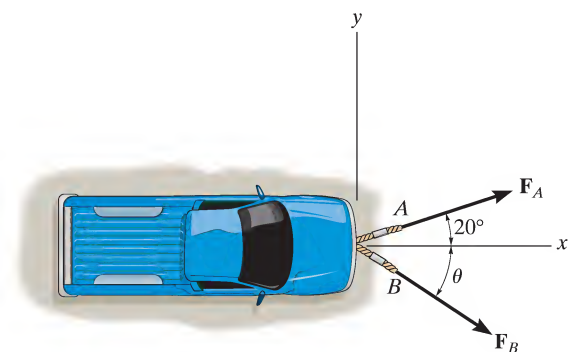
$$\frac{F_B}{\sin 20^\circ} = \frac{950}{\sin 110^\circ}$$

$$F_B = 346 \text{ N}$$

Ans



**2–19.** The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive  $x$  axis, determine the magnitudes of forces  $F_A$  and  $F_B$  acting on each rope and the angle  $\theta$  of  $F_B$  so that the magnitude of  $F_B$  is a *minimum*.  $F_A$  acts at  $20^\circ$  from the  $x$  axis as shown.



**Parallelogram Law :** In order to produce a *minimum* force  $F_B$ ,  $F_B$  has to act perpendicular to  $F_A$ . The parallelogram law of addition is shown in Fig. (a).

**Trigonometry :** Fig. (b).

$$F_B = 950 \sin 20^\circ = 325 \text{ N}$$

Ans

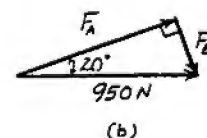
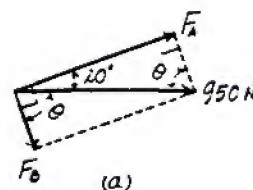
$$F_A = 950 \cos 20^\circ = 893 \text{ N}$$

Ans

The angle  $\theta$  is

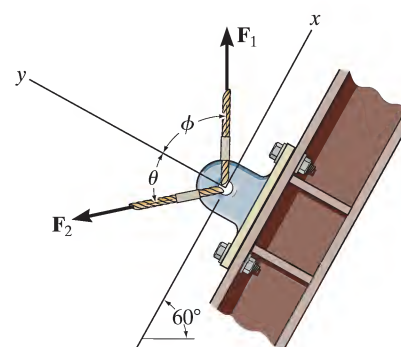
$$\theta = 90^\circ - 20^\circ = 70.0^\circ$$

Ans



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**\*2–20.** If  $\phi = 45^\circ$ ,  $F_1 = 5 \text{ kN}$ , and the resultant force is  $6 \text{ kN}$  directed along the positive  $y$  axis, determine the required magnitude of  $F_2$  and its direction  $\theta$ .



The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_2 = \sqrt{6^2 + 5^2 - 2(6)(5) \cos 45^\circ}$$

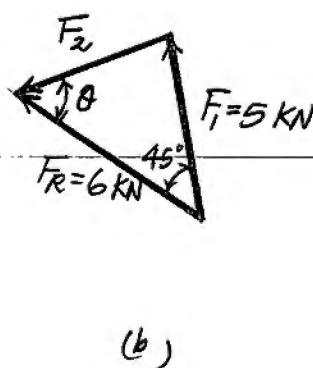
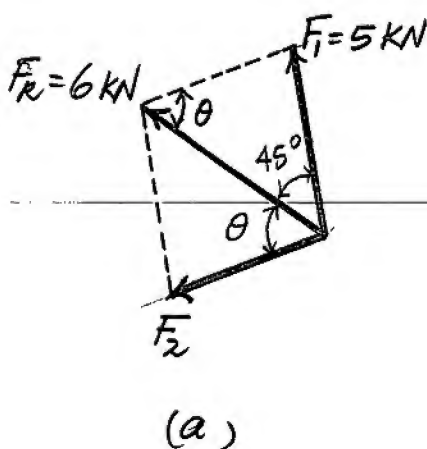
$$= 4.310 \text{ kN} = 4.31 \text{ kN}$$

**Ans.**

Using this result and applying the law of sines to Fig. *b*, yields

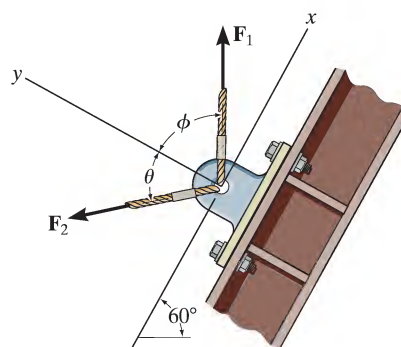
$$\frac{\sin \theta}{5} = \frac{\sin 45^\circ}{4.310}$$

$$\theta = 55.1^\circ \text{ Ans.}$$



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•2–21. If  $\phi = 30^\circ$  and the resultant force is to be 6 kN directed along the positive y axis, determine the magnitudes of  $F_1$  and  $F_2$  and the angle  $\theta$  if  $F_2$  is required to be a minimum.



For  $F_2$  to be minimum, it has to be directed perpendicular to  $F_R$ .

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

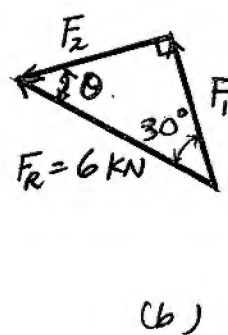
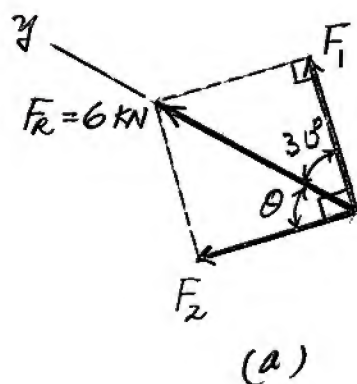
By applying simple trigonometry to Fig. *b*,

$$F_1 = 6 \cos 30^\circ = 5.20 \text{ kN} \quad \text{Ans.}$$

$$F_2 = 6 \sin 30^\circ = 3 \text{ kN} \quad \text{Ans.}$$

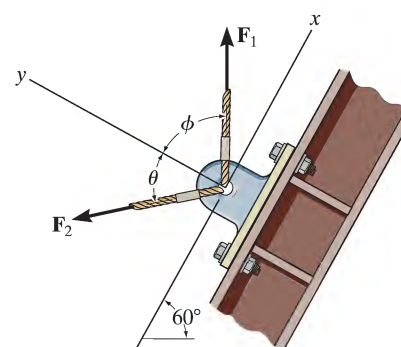
and

$$\theta = 90^\circ - 30^\circ = 60^\circ \quad \text{Ans.}$$



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**2-22.** If  $\phi = 30^\circ$ ,  $F_1 = 5 \text{ kN}$ , and the resultant force is to be directed along the positive  $y$  axis, determine the magnitude of the resultant force if  $F_2$  is to be a minimum. Also, what is  $F_2$  and the angle  $\theta$ ?



**Parallelogram Law and Triangular Rule:** The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

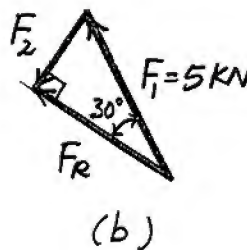
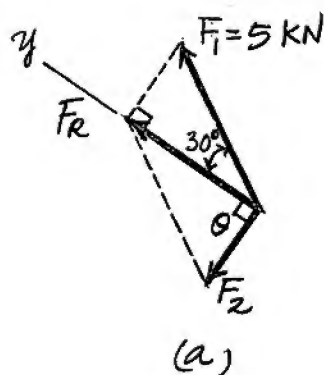
For  $F_2$  to be minimum, it must be directed perpendicular to the resultant force. Thus,

$$\theta = 90^\circ \quad \text{Ans.}$$

By applying simple trigonometry to Fig. *b*,

$$F_2 = 5 \sin 30^\circ = 2.50 \text{ kN} \quad \text{Ans.}$$

$$F_R = 5 \cos 30^\circ = 4.33 \text{ kN} \quad \text{Ans.}$$



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2-23. If  $\theta = 30^\circ$  and  $F_2 = 6$  kN, determine the magnitude of the resultant force acting on the plate and its direction measured clockwise from the positive  $x$  axis.

**Parallelogram Law and Triangular Rule:** This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelogram.

**Determination of Unknowns:** Referring to Fig. *b*,  $F'$  and  $\alpha$  can be determined as follows.

$$F' = \sqrt{4^2 + 5^2} = 6.403 \text{ kN}$$

$$\tan \alpha = \frac{5}{4} \quad \alpha = 51.34^\circ$$

Using the results for  $F'$  and  $\alpha$  and referring to Fig. *c*,  $F_R$  and  $\beta$  can be determined.

$$F_R = \sqrt{6^2 + 6.403^2 - 2(6)(6.403)\cos(51.34^\circ + 30^\circ)}$$

$$= 8.089 \text{ kN} = 8.09 \text{ kN}$$

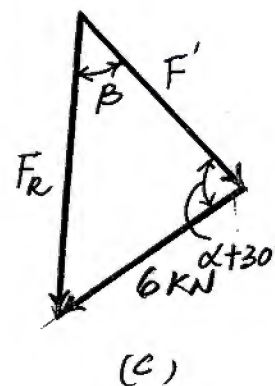
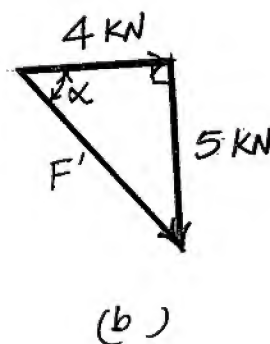
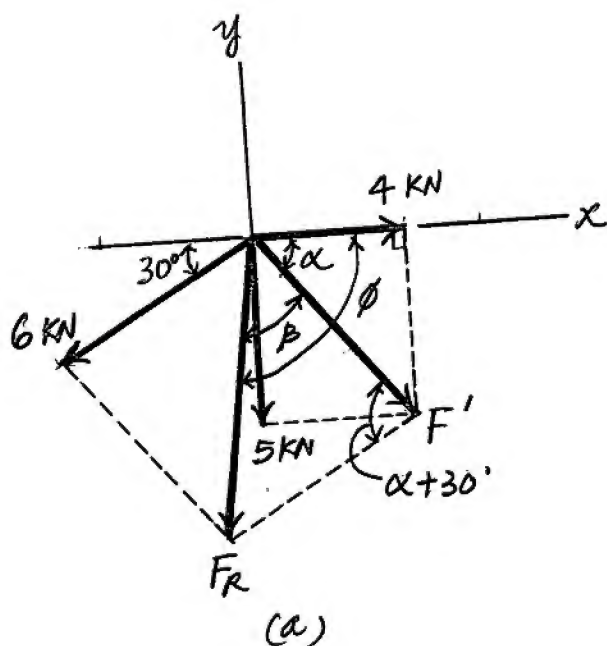
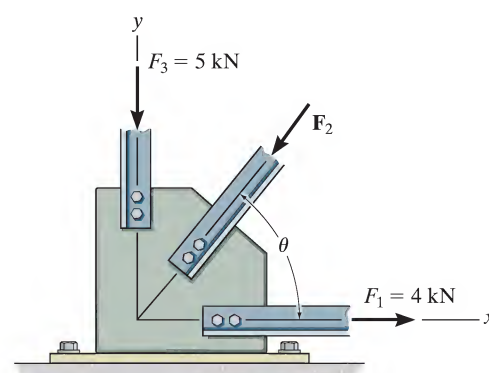
Ans.

$$\frac{\sin \beta}{6} = \frac{\sin(51.34^\circ + 30^\circ)}{8.089} \quad \beta = 47.16^\circ$$

Thus, the direction angle  $\phi$  of  $F_R$ , measured clockwise from the positive  $x$  axis, is

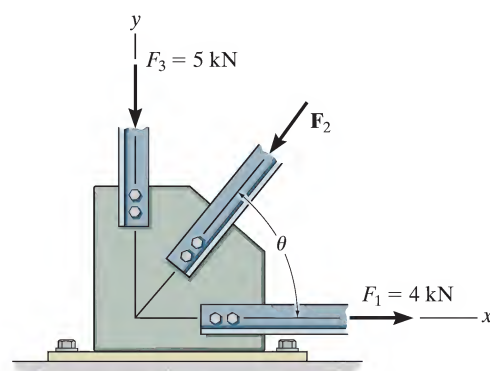
$$\phi = \alpha + \beta = 51.34^\circ + 47.16^\circ = 98.5^\circ$$

Ans.



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\*2-24. If the resultant force  $\mathbf{F}_R$  is directed along a line measured  $75^\circ$  clockwise from the positive  $x$  axis and the magnitude of  $\mathbf{F}_2$  is to be a minimum, determine the magnitudes of  $\mathbf{F}_R$  and  $\mathbf{F}_2$  and the angle  $\theta \leq 90^\circ$ .



This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelograms. For  $\mathbf{F}_1$  to be minimum, it must be perpendicular to the resultant force's line of action. Thus,

$$\theta = 90^\circ - 75^\circ = 15^\circ$$

Ans.

Referring to Fig. *b*,  $F'$  and  $\alpha$  can be determined.

$$F' = \sqrt{4^2 + 5^2} = 6.403 \text{ kN}$$

$$\tan \alpha = \frac{5}{4} \quad \alpha = 51.34^\circ$$

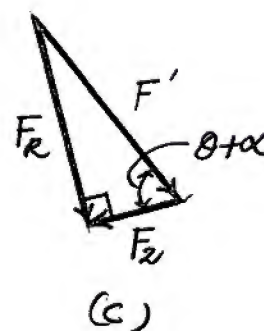
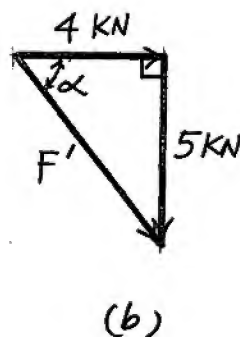
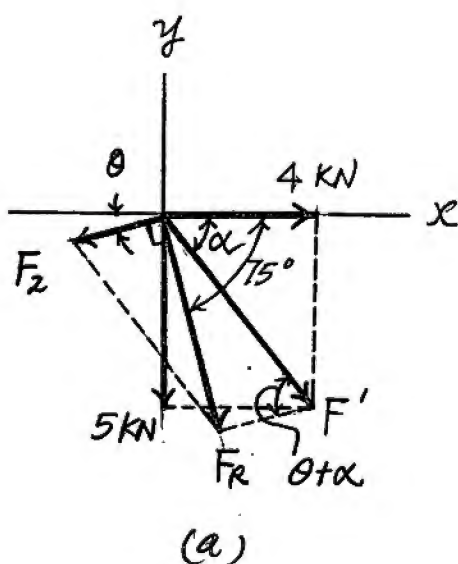
Using the results for  $\theta$ ,  $\alpha$ , and  $F'$ ,  $\mathbf{F}_R$  and  $\mathbf{F}_2$  can be determined by referring to Fig. *c*.

$$F_2 = 6.403 \cos(15^\circ + 51.43^\circ) = 2.57 \text{ kN}$$

Ans.

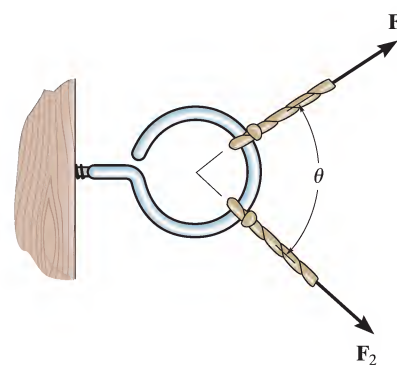
$$F_R = 6.403 \sin(15^\circ + 51.43^\circ) = 5.86 \text{ kN}$$

Ans.



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•2–25. Two forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  act on the screw eye. If their lines of action are at an angle  $\theta$  apart and the magnitude of each force is  $F_1 = F_2 = F$ , determine the magnitude of the resultant force  $\mathbf{F}_R$  and the angle between  $\mathbf{F}_R$  and  $\mathbf{F}_1$ .



$$\frac{F}{\sin \phi} = \frac{F}{\sin(\theta - \phi)}$$

$$\sin(\theta - \phi) = \sin \phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \quad \text{Ans}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F) \cos(180^\circ - \theta)}$$

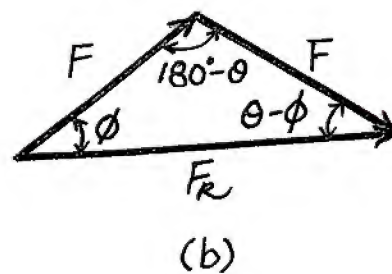
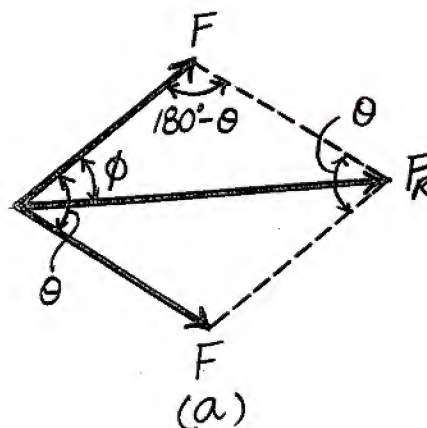
$$\text{Since } \cos(180^\circ - \theta) = -\cos \theta$$

$$F_R = F(\sqrt{2})\sqrt{1 + \cos \theta}$$

$$\text{Since } \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos \theta}{2}}$$

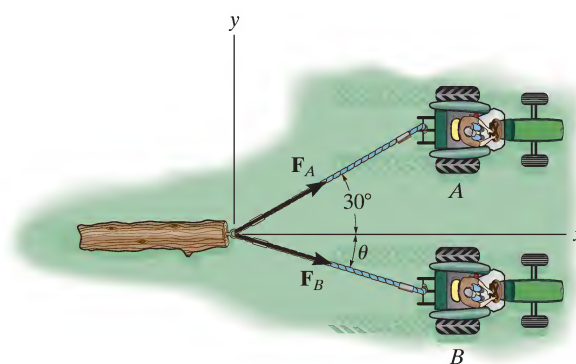
Then

$$F_R = 2F \cos\left(\frac{\theta}{2}\right) \quad \text{Ans}$$



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2-26. The log is being towed by two tractors  $A$  and  $B$ . Determine the magnitudes of the two towing forces  $F_A$  and  $F_B$  if it is required that the resultant force have a magnitude  $F_R = 10$  kN and be directed along the  $x$  axis. Set  $\theta = 15^\circ$ .



**Parallelogram Law** : The parallelogram law of addition is shown in Fig. (a).

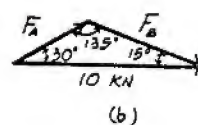
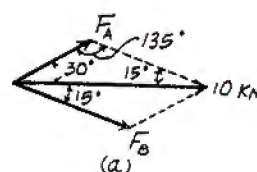
**Trigonometry** : Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 15^\circ} = \frac{10}{\sin 135^\circ}$$

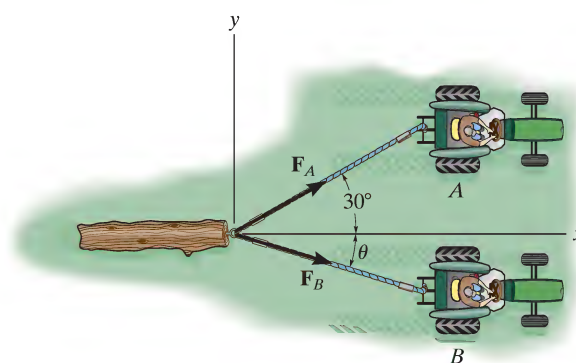
$$F_A = 3.66 \text{ kN} \quad \text{Ans}$$

$$\frac{F_B}{\sin 30^\circ} = \frac{10}{\sin 135^\circ}$$

$$F_B = 7.07 \text{ kN} \quad \text{Ans}$$



2-27. The resultant  $F_R$  of the two forces acting on the log is to be directed along the positive  $x$  axis and have a magnitude of 10 kN, determine the angle  $\theta$  of the cable, attached to  $B$  such that the magnitude of force  $F_B$  in this cable is a minimum. What is the magnitude of the force in each cable for this situation?



**Parallelogram Law** : In order to produce a *minimum* force  $F_B$ ,  $F_B$  has to act perpendicular to  $F_A$ . The parallelogram law of addition is shown in Fig. (a).

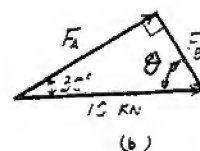
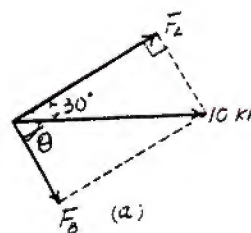
**Trigonometry** : Fig. (b).

$$F_B = 10 \sin 30^\circ = 5.00 \text{ kN} \quad \text{Ans}$$

$$F_A = 10 \cos 30^\circ = 8.66 \text{ kN} \quad \text{Ans}$$

The angle  $\theta$  is

$$\theta = 90^\circ - 30^\circ = 60.0^\circ \quad \text{Ans}$$

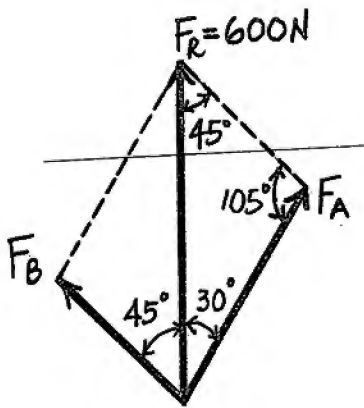


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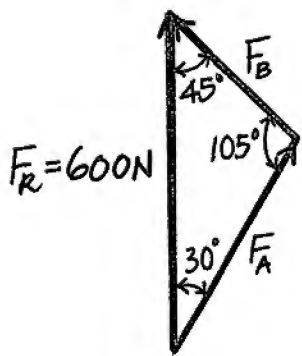
\*2-28. The beam is to be hoisted using two chains. Determine the magnitudes of forces  $F_A$  and  $F_B$  acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set  $\theta = 45^\circ$ .

$$\frac{F_A}{\sin 45^\circ} = \frac{600}{\sin 105^\circ}; \quad F_A = 439 \text{ N} \quad \text{Ans}$$

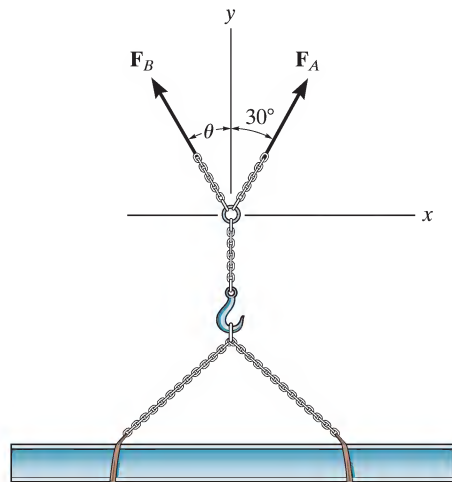
$$\frac{F_B}{\sin 30^\circ} = \frac{600}{\sin 105^\circ}; \quad F_B = 311 \text{ N} \quad \text{Ans}$$



(a)

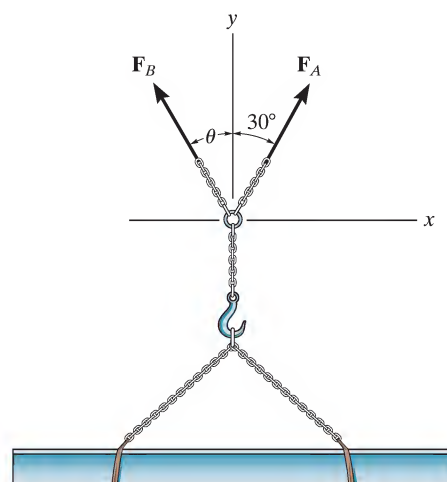


(b)



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•2–29. The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive  $y$  axis, determine the magnitudes of forces  $F_A$  and  $F_B$  acting on each chain and the angle  $\theta$  of  $F_B$  so that the magnitude of  $F_B$  is a *minimum*.  $F_A$  acts at  $30^\circ$  from the  $y$  axis, as shown.

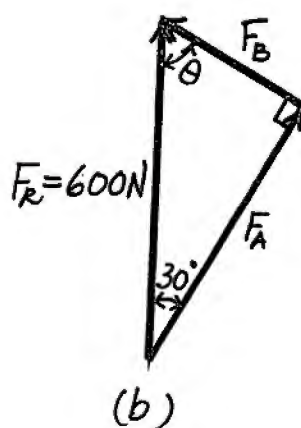
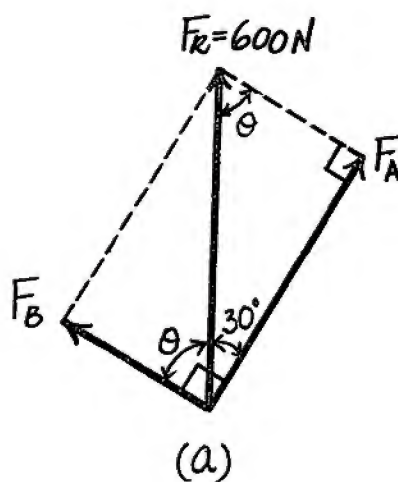


For minimum  $F_B$ , require

$$\theta = 60^\circ \quad \text{Ans}$$

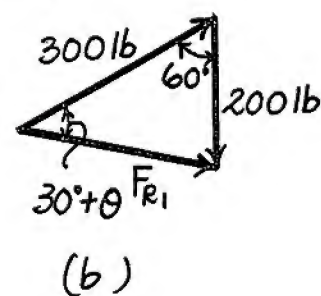
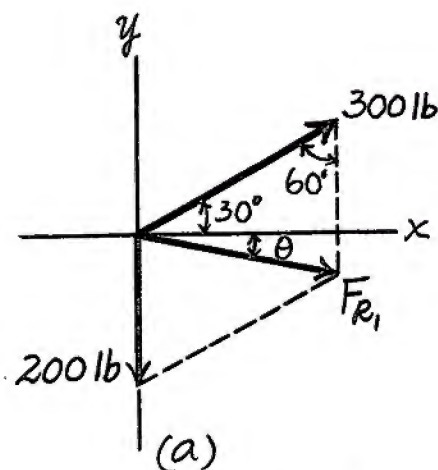
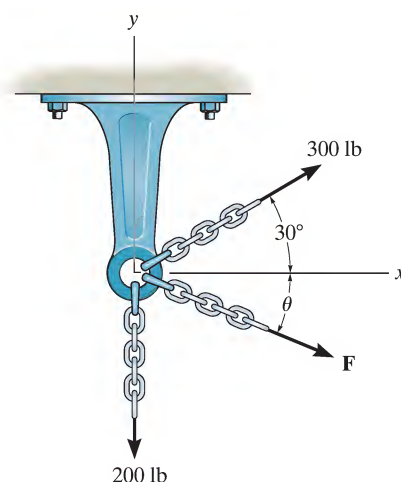
$$F_A = 600 \cos 30^\circ = 520 \text{ N} \quad \text{Ans}$$

$$F_B = 600 \sin 30^\circ = 300 \text{ N} \quad \text{Ans}$$



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**2–30.** Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle  $\theta$  of the third chain measured clockwise from the positive  $x$  axis, so that the magnitude of force  $\mathbf{F}$  in this chain is a *minimum*. All forces lie in the  $x$ - $y$  plane. What is the magnitude of  $\mathbf{F}$ ? *Hint:* First find the resultant of the two known forces. Force  $\mathbf{F}$  acts in this direction.



Cosine law :

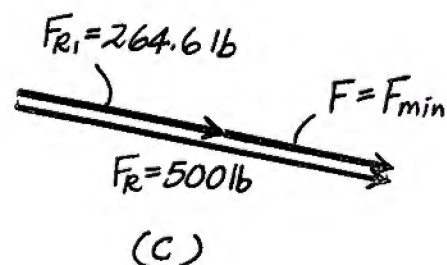
$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$

Sine law :

$$\frac{\sin(30^\circ + \theta)}{200} = \frac{\sin 60^\circ}{264.6} \quad \theta = 10.9^\circ \quad \text{Ans}$$

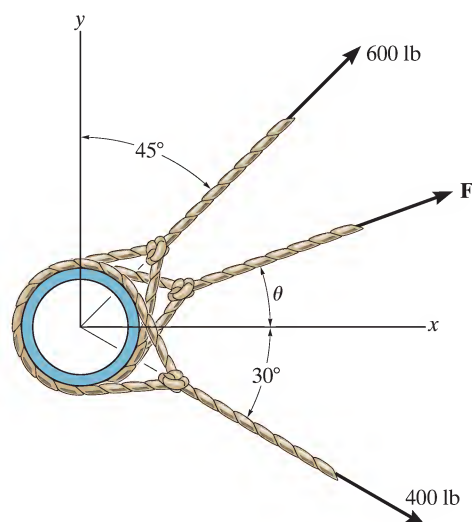
When  $\mathbf{F}$  is directed along  $\mathbf{F}_{R1}$ ,  $\mathbf{F}$  will be minimum to create the resultant force.

$$\begin{aligned} F_R &= F_{R1} + F \\ 500 &= 264.6 + F_{min} \\ F_{min} &= 235 \text{ lb} \end{aligned} \quad \text{Ans}$$



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**2-31.** Three cables pull on the pipe such that they create a resultant force having a magnitude of 900 lb. If two of the cables are subjected to known forces, as shown in the figure, determine the angle  $\theta$  of the third cable so that the magnitude of force  $F$  in this cable is a *minimum*. All forces lie in the  $x$ - $y$  plane. What is the magnitude of  $F$ ? *Hint:* First find the resultant of the two known forces.

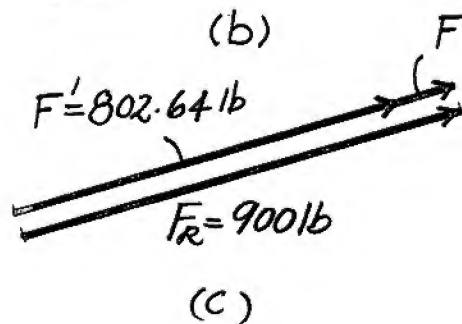
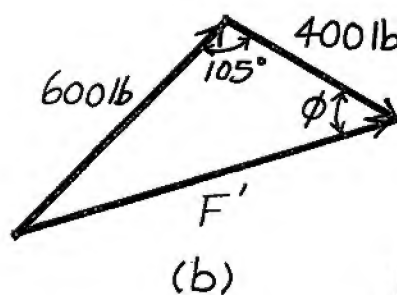
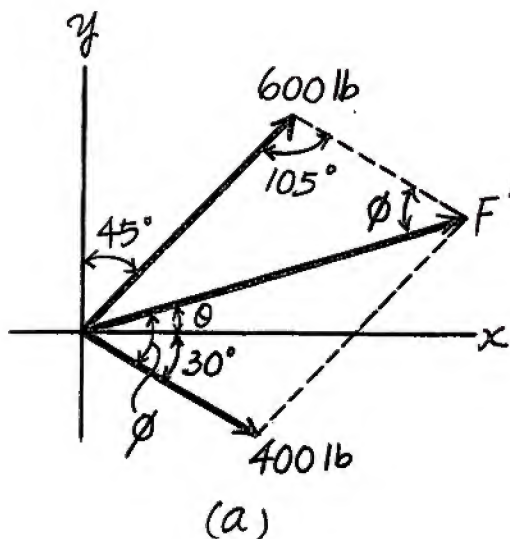


$$F' = \sqrt{(600)^2 + (400)^2 - 2(600)(400)\cos 105^\circ} = 802.64 \text{ lb}$$

$$F = 900 - 802.64 = 97.4 \text{ lb} \quad \text{Ans}$$

$$\frac{\sin \phi}{600} = \frac{\sin 105^\circ}{802.64}; \quad \phi = 46.22^\circ$$

$$\theta = 46.22^\circ - 30^\circ = 16.2^\circ \quad \text{Ans}$$



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**\*2–32.** Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive  $x$  axis.

**Rectangular Components:** By referring to Fig.  $a$ , the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ , and  $F_3$  can be written as

$$(F_1)_x = 30 \cos 45^\circ = 21.21 \text{ lb}$$

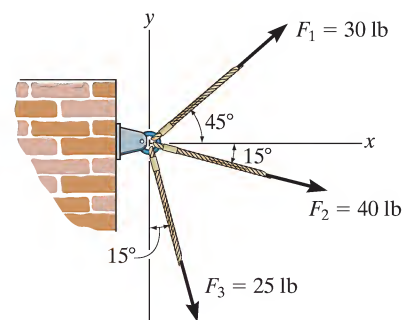
$$(F_1)_y = 30 \sin 45^\circ = 21.21 \text{ lb}$$

$$(F_2)_x = 40 \cos 15^\circ = 38.64 \text{ lb}$$

$$(F_2)_y = 40 \sin 15^\circ = 10.35 \text{ lb}$$

$$(F_3)_x = 25 \sin 15^\circ = 6.47 \text{ lb}$$

$$(F_3)_y = 25 \cos 15^\circ = 24.15 \text{ lb}$$



**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$+\rightarrow \Sigma (F_R)_x = \Sigma F_x; \quad (F_R)_x = 21.21 + 38.64 + 6.47 = 66.32 \text{ lb} \rightarrow$$

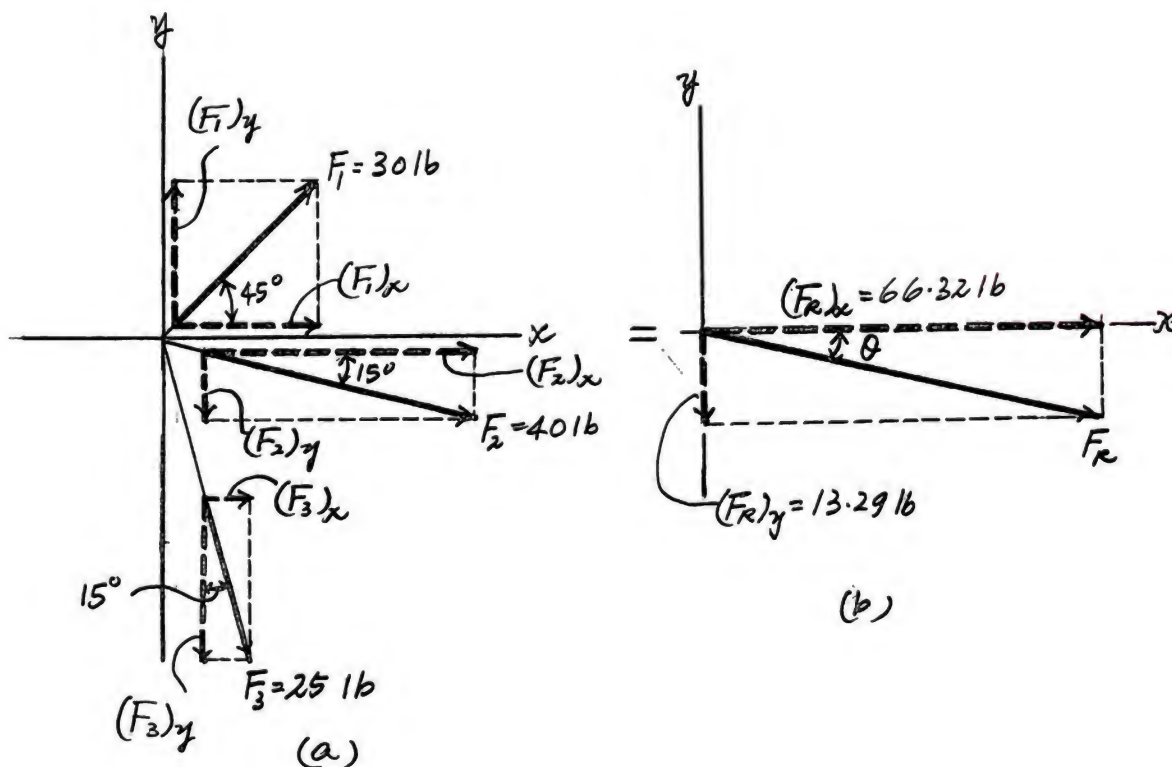
$$+\uparrow \Sigma (F_R)_y = \Sigma F_y; \quad (F_R)_y = 21.21 - 10.35 - 24.15 = -13.29 \text{ lb} = 13.29 \text{ lb} \downarrow$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{66.32^2 + 13.29^2} = 67.6 \text{ lb} \quad \text{Ans.}$$

The direction angle  $\theta$  of  $F_R$ , measured clockwise from the positive  $x$  axis, is

$$\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left( \frac{13.29}{66.32} \right) = 11.3^\circ \quad \text{Ans.}$$



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•2–33. If  $F_1 = 600\text{ N}$  and  $\phi = 30^\circ$ , determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive  $x$  axis.

**Rectangular Components:** By referring to Fig. *a*, the  $x$  and  $y$  components of each force can be written as

$$\begin{aligned}(F_1)_x &= 600 \cos 30^\circ = 519.62\text{ N} & (F_1)_y &= 600 \sin 30^\circ = 300\text{ N} \\ (F_2)_x &= 500 \cos 60^\circ = 250\text{ N} & (F_2)_y &= 500 \sin 60^\circ = 433.0\text{ N} \\ (F_3)_x &= 450 \left(\frac{3}{5}\right) = 270\text{ N} & (F_3)_y &= 450 \left(\frac{4}{5}\right) = 360\text{ N}\end{aligned}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned}+\rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 519.62 + 250 - 270 = 499.62\text{ N} \rightarrow \\ +\uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= 300 - 433.01 - 360 = -493.01\text{ N} = 493.01\text{ N} \downarrow\end{aligned}$$

The magnitude of the resultant force  $F_R$  is

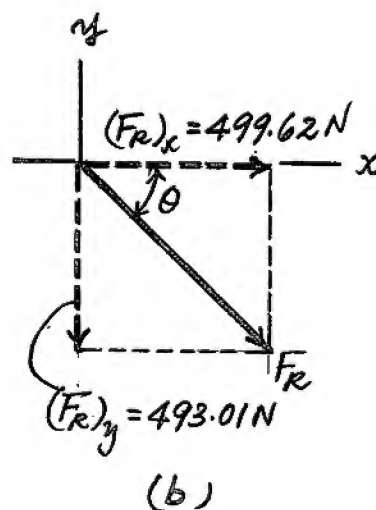
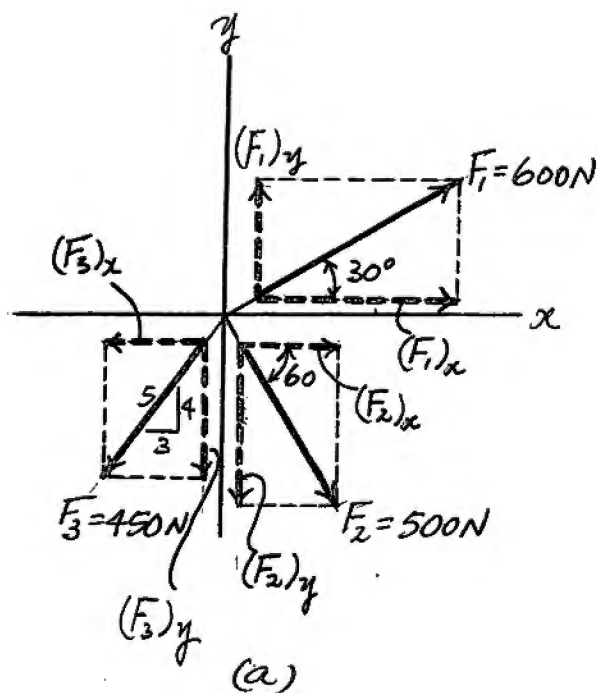
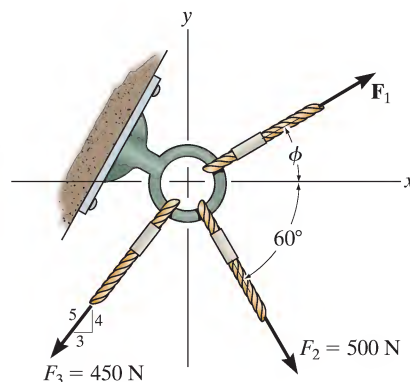
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91\text{ N} = 702\text{ N}$$

Ans.

The direction angle  $\theta$  of  $F_R$ , Fig. *b*, measured clockwise from the  $x$  axis, is

$$\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left( \frac{493.01}{499.62} \right) = 44.6^\circ$$

Ans.

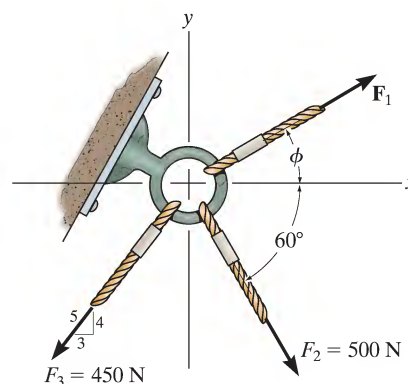


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2-34. If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive  $x$  axis is  $\theta = 30^\circ$ , determine the magnitude of  $F_1$  and the angle  $\phi$ .

**Rectangular Components:** By referring to Figs. *a* and *b*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_R$  can be written as

$$\begin{aligned} (F_1)_x &= F_1 \cos \phi & (F_1)_y &= F_1 \sin \phi \\ (F_2)_x &= 500 \cos 60^\circ = 250 \text{ N} & (F_2)_y &= 500 \sin 60^\circ = 433.01 \text{ N} \\ (F_3)_x &= 450 \left( \frac{3}{5} \right) = 270 \text{ N} & (F_3)_y &= 450 \left( \frac{4}{5} \right) = 360 \text{ N} \\ (F_R)_x &= 600 \cos 30^\circ = 519.62 \text{ N} & (F_R)_y &= 600 \sin 30^\circ = 300 \text{ N} \end{aligned}$$



**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned} + \rightarrow \Sigma (F_R)_x &= \Sigma F_x; & 519.62 &= F_1 \cos \phi + 250 - 270 \\ & & F_1 \cos \phi &= 539.62 \end{aligned} \quad (1)$$

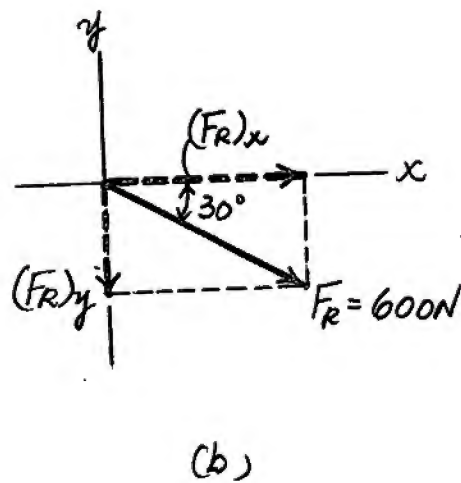
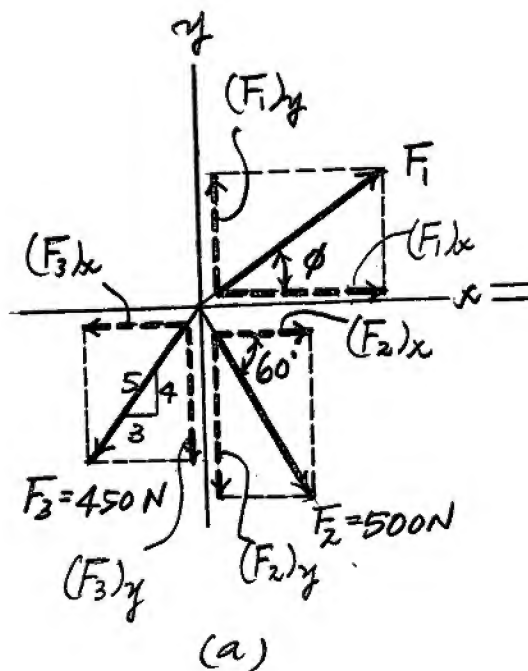
$$\begin{aligned} + \uparrow \Sigma (F_R)_y &= \Sigma F_y; & -300 &= F_1 \sin \phi - 433.01 - 360 \\ & & F_1 \sin \phi &= 493.01 \end{aligned} \quad (2)$$

Solving Eqs. (1) and (2), yields

$$\phi = 42.4^\circ$$

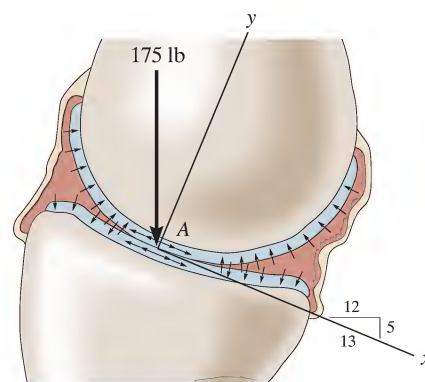
$$F_1 = 731 \text{ N}$$

Ans.



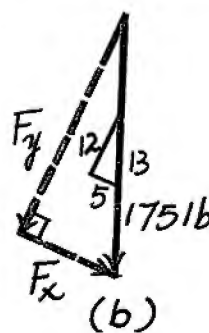
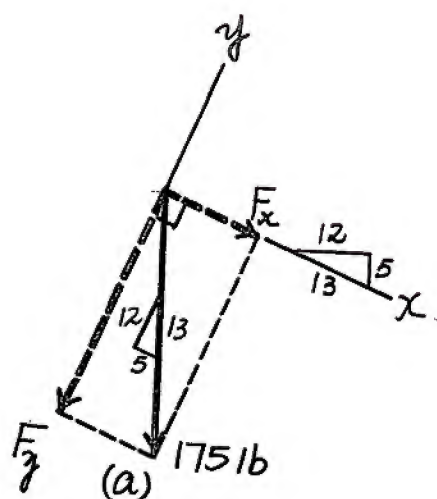
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**2-35.** The contact point between the femur and tibia bones of the leg is at *A*. If a vertical force of 175 lb is applied at this point, determine the components along the *x* and *y* axes. Note that the *y* component represents the normal force on the load-bearing region of the bones. Both the *x* and *y* components of this force cause synovial fluid to be squeezed out of the bearing space.



$$F_x = 175 \left( \frac{5}{13} \right) = 67.3 \text{ lb} \quad \text{Ans}$$

$$F_y = -175 \left( \frac{12}{13} \right) = -162 \text{ lb} \quad \text{Ans}$$



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\*2-36. If  $\phi = 30^\circ$  and  $F_2 = 3$  kN, determine the magnitude of the resultant force acting on the plate and its direction  $\theta$  measured clockwise from the positive  $x$  axis.

**Rectangular Components:** By referring to Fig. *a*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ , and  $F_3$  can be written as

$$(F_1)_x = 4 \sin 30^\circ = 2 \text{ kN}$$

$$(F_1)_y = 4 \cos 30^\circ = 3.464 \text{ kN}$$

$$(F_2)_x = 3 \cos 30^\circ = 2.598 \text{ kN}$$

$$(F_2)_y = 3 \sin 30^\circ = 1.50 \text{ kN}$$

$$(F_3)_x = 5 \left( \frac{4}{5} \right) = 4 \text{ kN}$$

$$(F_3)_y = 5 \left( \frac{3}{5} \right) = 3 \text{ kN}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$+\rightarrow \Sigma (F_R)_x = \Sigma F_x; (F_R)_x = -2 + 2.598 + 4 = 4.598 \text{ kN} \rightarrow$$

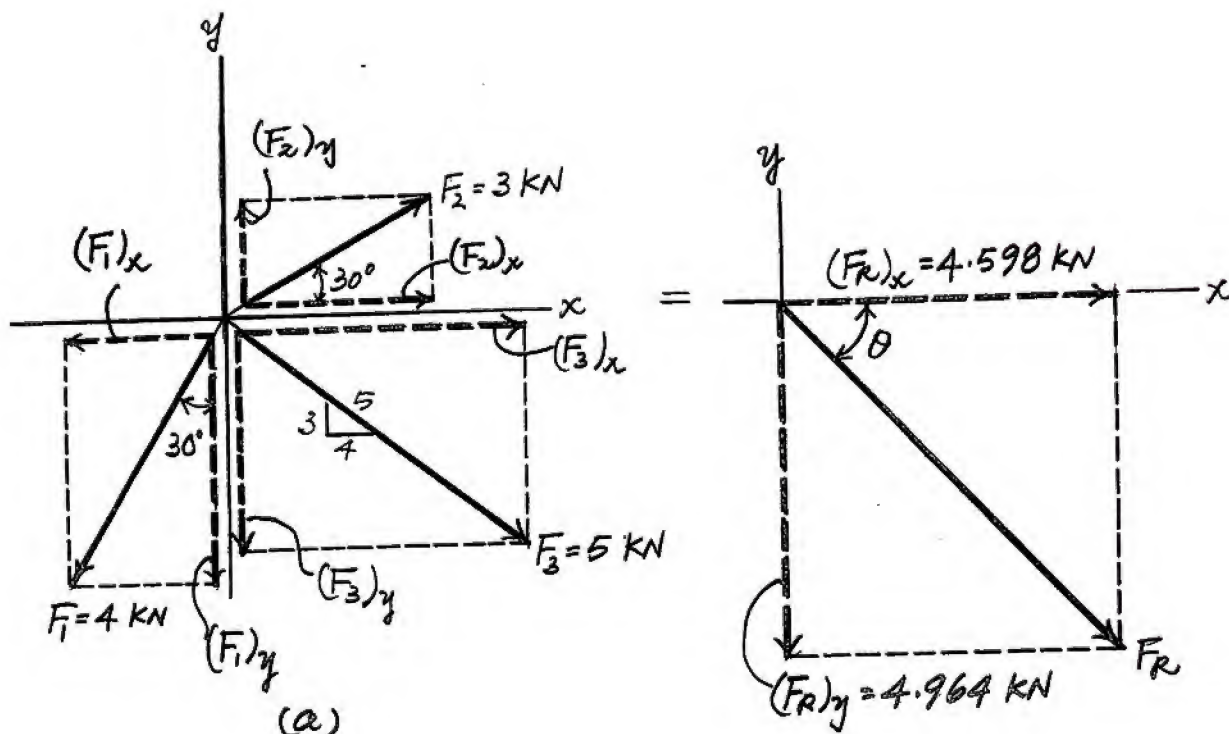
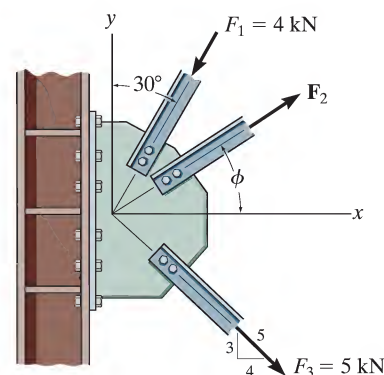
$$+\uparrow \Sigma (F_R)_y = \Sigma F_y; (F_R)_y = -3.464 + 1.50 - 3 = -4.964 \text{ kN} = 4.964 \text{ kN} \downarrow$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{4.598^2 + 4.964^2} = 6.77 \text{ kN} \quad \text{Ans.}$$

The direction angle  $\theta$  of  $F_R$ , Fig. *b*, measured clockwise from the positive  $x$  axis, is

$$\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left( \frac{4.964}{4.598} \right) = 47.2^\circ \quad \text{Ans.}$$



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•2–37. If the magnitude for the resultant force acting on the plate is required to be 6 kN and its direction measured clockwise from the positive  $x$  axis is  $\theta = 30^\circ$ , determine the magnitude of  $F_2$  and its direction  $\phi$ .

**Rectangular Components:** By referring to Figs. *a* and *b*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_R$  can be written as

$$(F_1)_x = 4 \sin 30^\circ = 2 \text{ kN}$$

$$(F_2)_x = F_2 \cos \phi$$

$$(F_3)_x = 5 \left( \frac{4}{5} \right) = 4 \text{ kN}$$

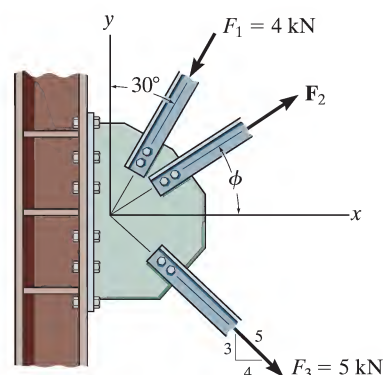
$$(F_R)_x = 6 \cos 30^\circ = 5.196 \text{ kN}$$

$$(F_1)_y = 4 \cos 30^\circ = 3.464 \text{ kN}$$

$$(F_2)_y = F_2 \sin \phi$$

$$(F_3)_y = 5 \left( \frac{3}{5} \right) = 3 \text{ kN}$$

$$(F_R)_y = 6 \sin 30^\circ = 3 \text{ kN}$$



**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned} \rightarrow \Sigma(F_R)_x = \Sigma F_x; \quad 5.196 &= -2 + F_2 \cos \phi + 4 \\ F_2 \cos \phi &= 3.196 \end{aligned} \quad (1)$$

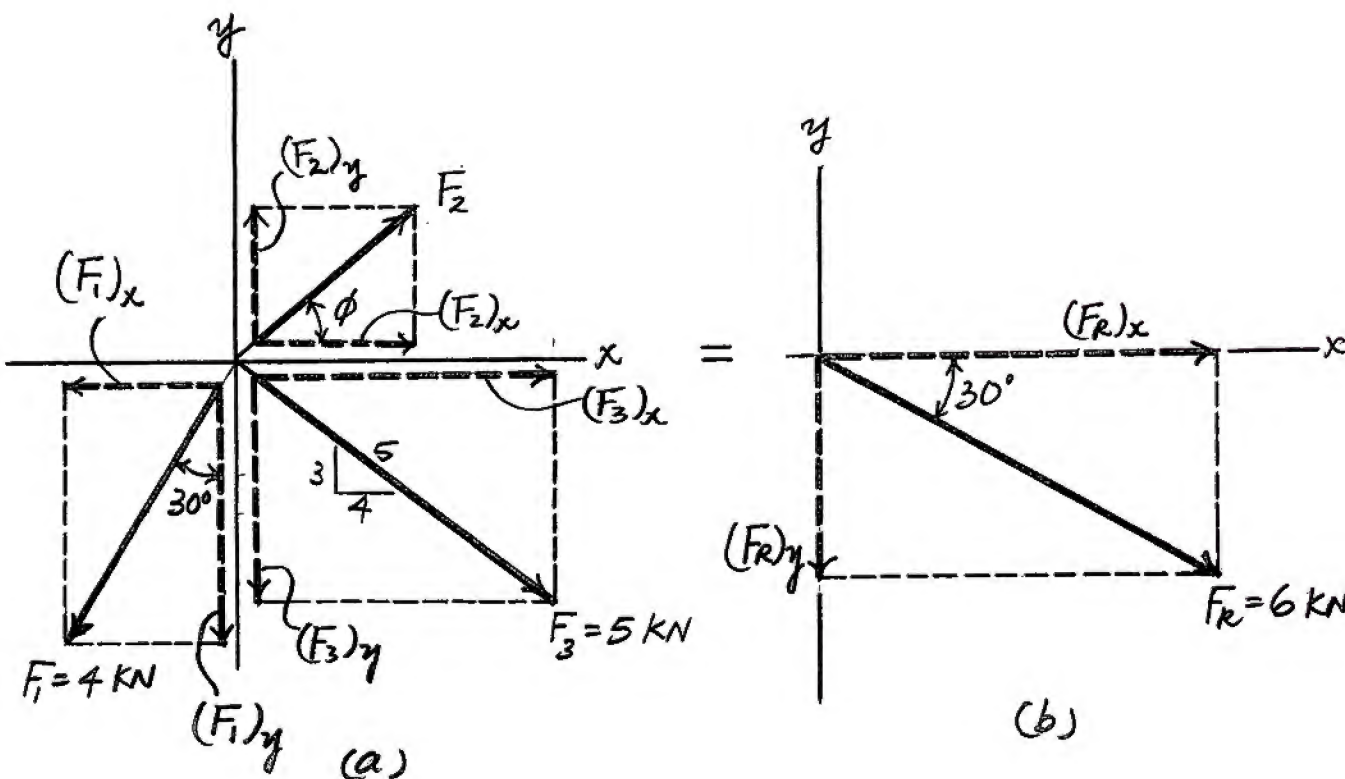
$$\begin{aligned} + \uparrow \Sigma(F_R)_y = \Sigma F_y; \quad -3 &= -3.464 + F_2 \sin \phi - 3 \\ F_2 \sin \phi &= 3.464 \end{aligned} \quad (2)$$

Solving Eqs. (1) and (2), yields

$$\phi = 47.3^\circ$$

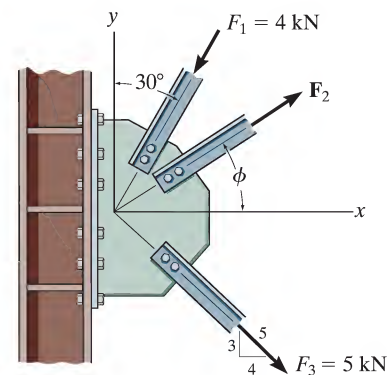
$$F_2 = 4.71 \text{ kN}$$

Ans.



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2-38. If  $\phi = 30^\circ$  and the resultant force acting on the gusset plate is directed along the positive  $x$  axis, determine the magnitudes of  $F_2$  and the resultant force.



**Rectangular Components:** By referring to Fig. *a*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_R$  can be written as

$$\begin{aligned}(F_1)_x &= 4 \sin 30^\circ = 2 \text{ kN} & (F_1)_y &= 4 \cos 30^\circ = 3.464 \text{ kN} \\(F_2)_x &= F_2 \cos 30^\circ = 0.8660 F_2 & (F_2)_y &= F_2 \sin 30^\circ = 0.5 F_2 \\(F_3)_x &= 5 \left( \frac{4}{5} \right) = 4 \text{ kN} & (F_3)_y &= 5 \left( \frac{3}{5} \right) = 3 \text{ kN} \\(F_R)_x &= F_R & (F_R)_y &= 0\end{aligned}$$

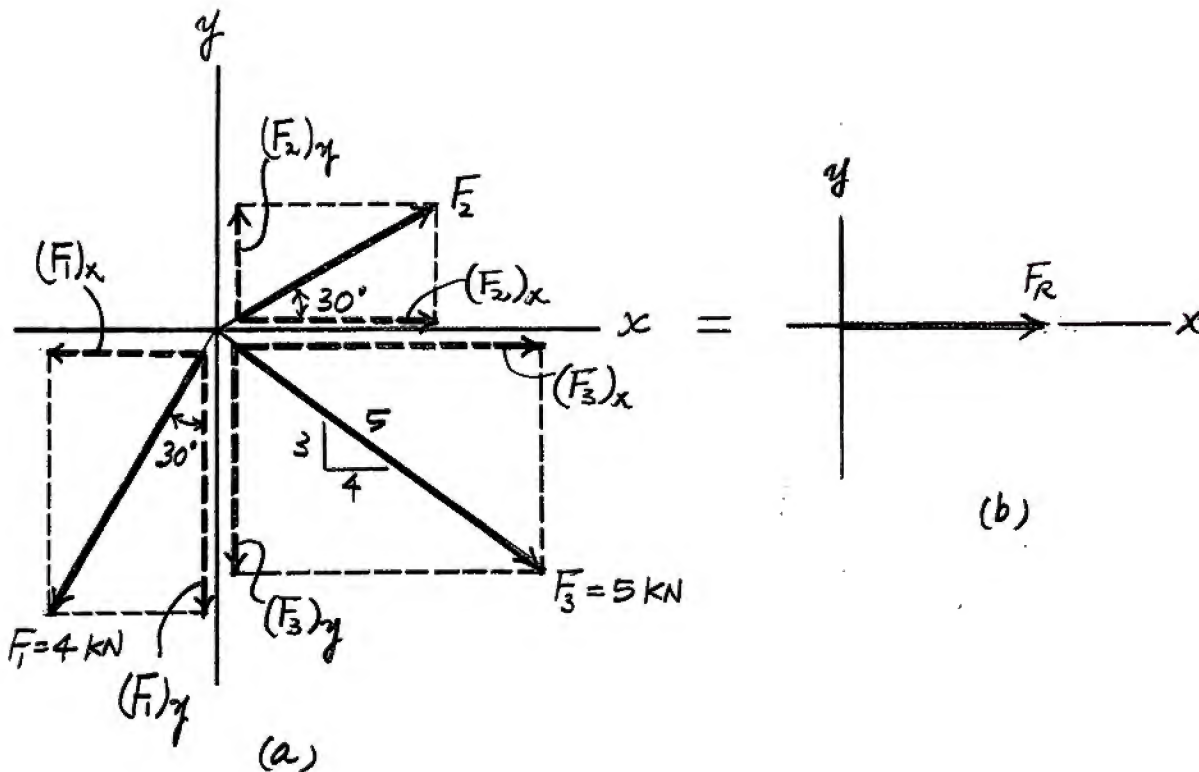
**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned}+\uparrow \Sigma (F_R)_y &= \Sigma F_y; \quad 0 = -3.464 + 0.5 F_2 - 3 \\F_2 &= 12.93 \text{ kN} \approx 12.9 \text{ kN}\end{aligned}$$

Ans.

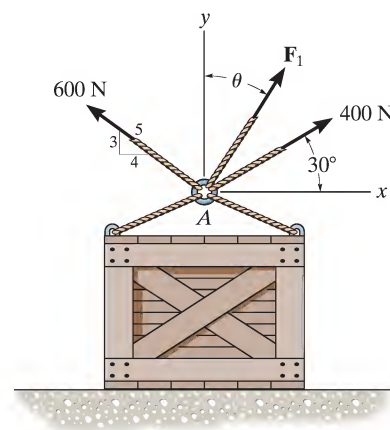
$$\begin{aligned}+\rightarrow \Sigma (F_R)_x &= \Sigma F_x; \quad F_R = -2 + 0.8660(12.93) + 4 \\&= 13.2 \text{ kN}\end{aligned}$$

Ans.



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**2–39.** Determine the magnitude of  $F_1$  and its direction  $\theta$  so that the resultant force is directed vertically upward and has a magnitude of 800 N.



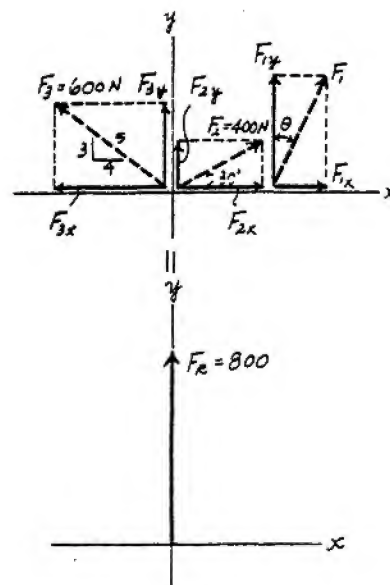
**Scalar Notation :** Summing the force components algebraically, we have

$$\rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} = 0 = F_1 \sin \theta + 400 \cos 30^\circ - 600 \left( \frac{4}{5} \right) \\ F_1 \sin \theta = 133.6 \quad [1]$$

$$+ \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 800 = F_1 \cos \theta + 400 \sin 30^\circ + 600 \left( \frac{3}{5} \right) \\ F_1 \cos \theta = 240 \quad [2]$$

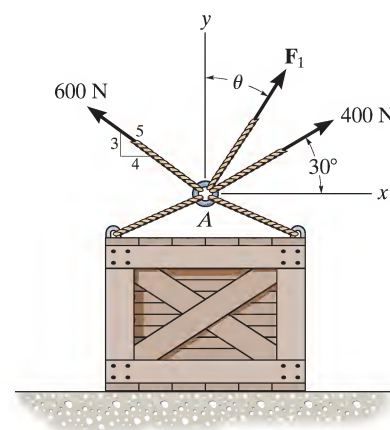
Solving Eq. [1] and [2] yields

$$\theta = 29.1^\circ \quad F_1 = 275 \text{ N} \quad \text{Ans}$$



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**\*2–40.** Determine the magnitude and direction measured counterclockwise from the positive  $x$  axis of the resultant force of the three forces acting on the ring  $A$ . Take  $F_1 = 500$  N and  $\theta = 20^\circ$ .



**Scalar Notation :** Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 500 \sin 20^\circ + 400 \cos 30^\circ - 600 \left( \frac{4}{5} \right) \\ &= 37.42 \text{ N } \rightarrow \end{aligned}$$

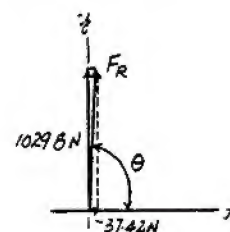
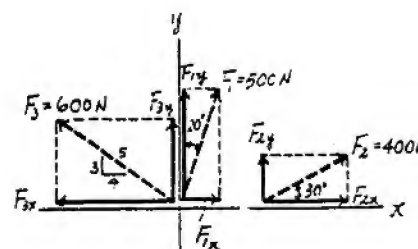
$$\begin{aligned} + \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} &= 500 \cos 20^\circ + 400 \sin 30^\circ + 600 \left( \frac{3}{5} \right) \\ &= 1029.8 \text{ N } \uparrow \end{aligned}$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN} \quad \text{Ans}$$

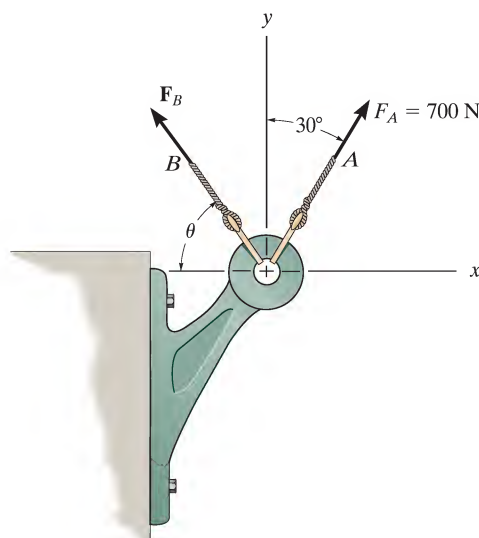
The directional angle  $\theta$  measured counterclockwise from positive  $x$  axis is

$$\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}} = \tan^{-1} \left( \frac{1029.8}{37.42} \right) = 87.9^\circ \quad \text{Ans}$$



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•2–41. Determine the magnitude and direction  $\theta$  of  $\mathbf{F}_B$  so that the resultant force is directed along the positive  $y$  axis and has a magnitude of 1500 N.



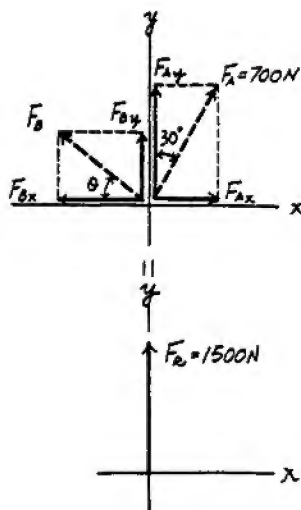
**Scalar Notation :** Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_x = \Sigma F_x; \quad 0 &= 700 \sin 30^\circ - F_B \cos \theta \\ F_B \cos \theta &= 350 \end{aligned} \quad [1]$$

$$\begin{aligned} + \uparrow F_y = \Sigma F_y; \quad 1500 &= 700 \cos 30^\circ + F_B \sin \theta \\ F_B \sin \theta &= 893.8 \end{aligned} \quad [2]$$

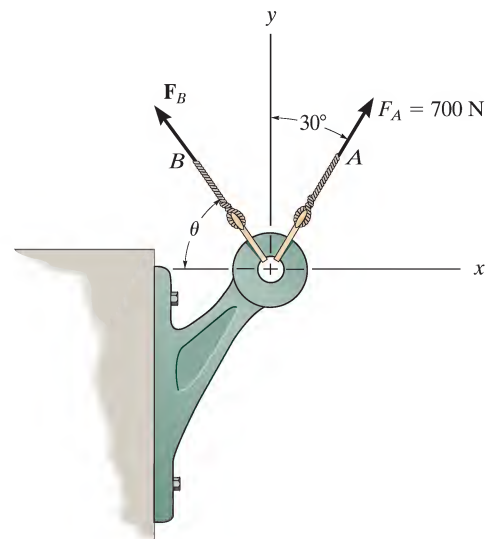
Solving Eq. [1] and [2] yields

$$\theta = 68.6^\circ \quad F_B = 960 \text{ N} \quad \text{Ans}$$



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**2–42.** Determine the magnitude and angle measured counterclockwise from the positive  $y$  axis of the resultant force acting on the bracket if  $F_B = 600$  N and  $\theta = 20^\circ$ .



**Scalar Notation :** Summing the force components algebraically, we have

$$\rightarrow F_{Rx} = \Sigma F_x: \quad F_{Rx} = 700 \sin 30^\circ - 600 \cos 20^\circ \\ = -213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

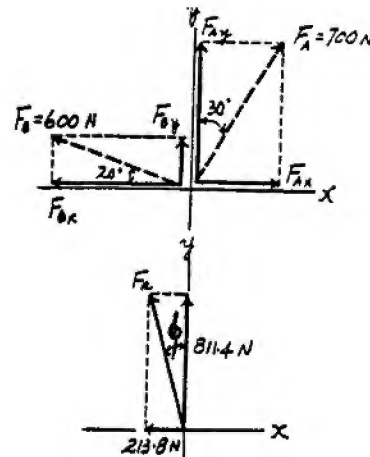
$$+ \uparrow F_{Ry} = \Sigma F_y: \quad F_{Ry} = 700 \cos 30^\circ + 600 \sin 20^\circ \\ = 811.4 \text{ N} \uparrow$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \quad \text{Ans}$$

The directional angle  $\theta$  measured counterclockwise from positive  $y$  axis is

$$\phi = \tan^{-1} \frac{F_{Rx}}{F_{Ry}} = \tan^{-1} \left( \frac{213.8}{811.4} \right) = 14.8^\circ \quad \text{Ans}$$



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2-43. If  $\phi = 30^\circ$  and  $F_1 = 250$  lb, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive  $x$  axis.

**Rectangular Components:** By referring to Fig. *a*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ , and  $F_3$  can be written as

$$\begin{aligned}(F_1)_x &= 250 \cos 30^\circ = 216.51 \text{ lb} & (F_1)_y &= 250 \sin 30^\circ = 125 \text{ lb} \\ (F_2)_x &= 300 \left(\frac{4}{5}\right) = 240 \text{ lb} & (F_2)_y &= 300 \left(\frac{3}{5}\right) = 180 \text{ lb} \\ (F_3)_x &= 260 \left(\frac{5}{13}\right) = 100 \text{ lb} & (F_3)_y &= 260 \left(\frac{12}{13}\right) = 240 \text{ lb}\end{aligned}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned}+\rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 216.51 + 240 - 100 = 356.51 \text{ lb} \rightarrow \\ +\uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= 125 - 180 - 240 = -295 \text{ lb} = 295 \text{ lb} \downarrow\end{aligned}$$

The magnitude of the resultant force  $F_R$  is

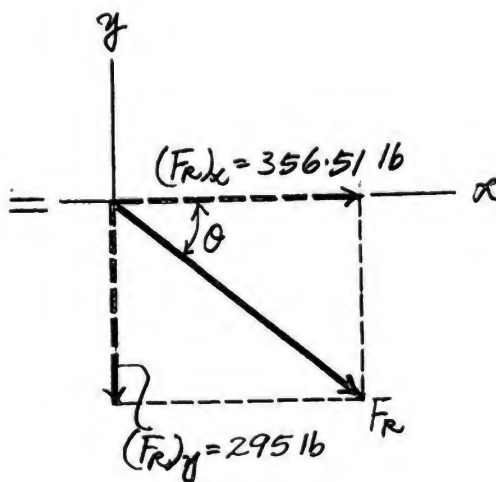
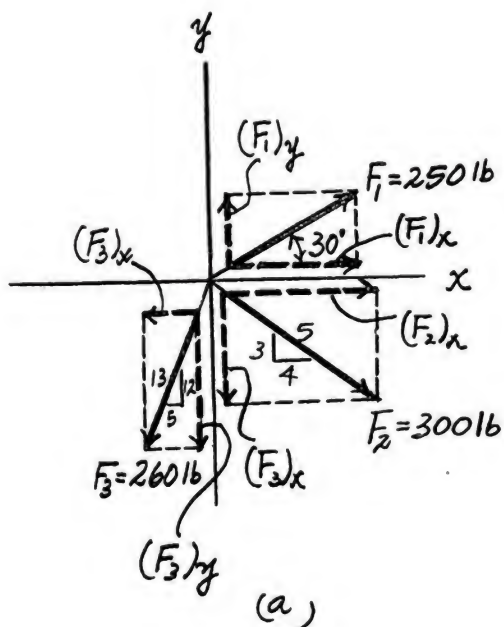
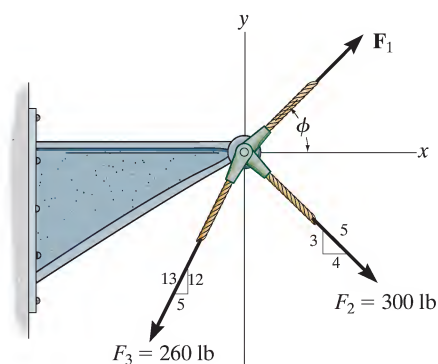
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{356.51^2 + 295^2} = 463 \text{ lb}$$

Ans.

The direction angle  $\theta$  of  $F_R$ , Fig. *b*, measured clockwise from the positive  $x$  axis, is

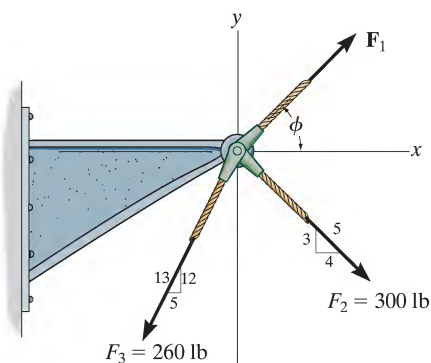
$$\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left( \frac{295}{356.51} \right) = 39.6^\circ$$

Ans.



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\*2-44. If the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive  $x$  axis, determine the magnitude of  $F_1$  and its direction  $\phi$ .



**Rectangular Components:** By referring to Fig.  $a$ , the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_R$  can be written as

$$\begin{aligned} (F_1)_x &= F_1 \cos \phi & (F_1)_y &= F_1 \sin \phi \\ (F_2)_x &= 300 \left( \frac{4}{5} \right) = 240 \text{ lb} & (F_2)_y &= 300 \left( \frac{3}{5} \right) = 180 \text{ lb} \\ (F_3)_x &= 260 \left( \frac{5}{13} \right) = 100 \text{ lb} & (F_3)_y &= 260 \left( \frac{12}{13} \right) = 240 \text{ lb} \\ (F_R)_x &= 400 \text{ lb} & (F_R)_y &= 0 \end{aligned}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

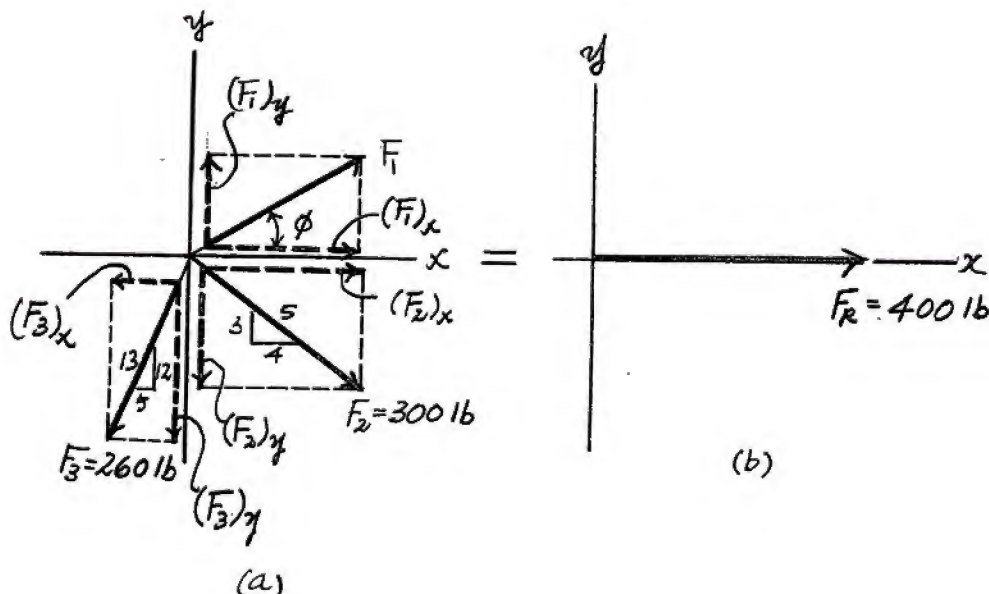
$$\begin{aligned} \rightarrow \Sigma (F_R)_x &= \Sigma F_x; & 400 &= F_1 \cos \phi + 240 - 100 \\ & & F_1 \cos \phi &= 260 \end{aligned} \quad (1)$$

$$\begin{aligned} + \uparrow \Sigma (F_R)_y &= \Sigma F_y; & 0 &= F_1 \sin \phi - 180 - 240 \\ & & F_1 \sin \phi &= 420 \end{aligned} \quad (2)$$

Solving Eqs. (1) and (2), yields

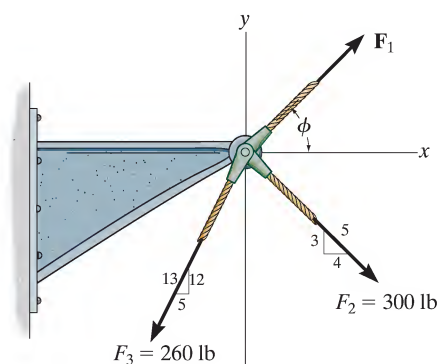
$$\phi = 58.2^\circ \quad F_1 = 494 \text{ lb}$$

Ans.



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•2–45. If the resultant force acting on the bracket is to be directed along the positive  $x$  axis and the magnitude of  $F_1$  is required to be a minimum, determine the magnitudes of the resultant force and  $F_1$ .



**Rectangular Components:** By referring to Figs.  $a$  and  $b$ , the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_R$  can be written as

$$\begin{aligned} (F_1)_x &= F_1 \cos \phi & (F_1)_y &= F_1 \sin \phi \\ (F_2)_x &= 300 \left( \frac{4}{5} \right) = 240 \text{ lb} & (F_2)_y &= 300 \left( \frac{3}{5} \right) = 180 \text{ lb} \\ (F_3)_x &= 260 \left( \frac{5}{13} \right) = 100 \text{ lb} & (F_3)_y &= 260 \left( \frac{12}{13} \right) = 240 \text{ lb} \\ (F_R)_x &= F_R & (F_R)_y &= 0 \end{aligned}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$+\uparrow \Sigma (F_R)_y = \Sigma F_y; \quad 0 = F_1 \sin \phi - 180 - 240$$

$$F_1 = \frac{420}{\sin \phi} \quad (1)$$

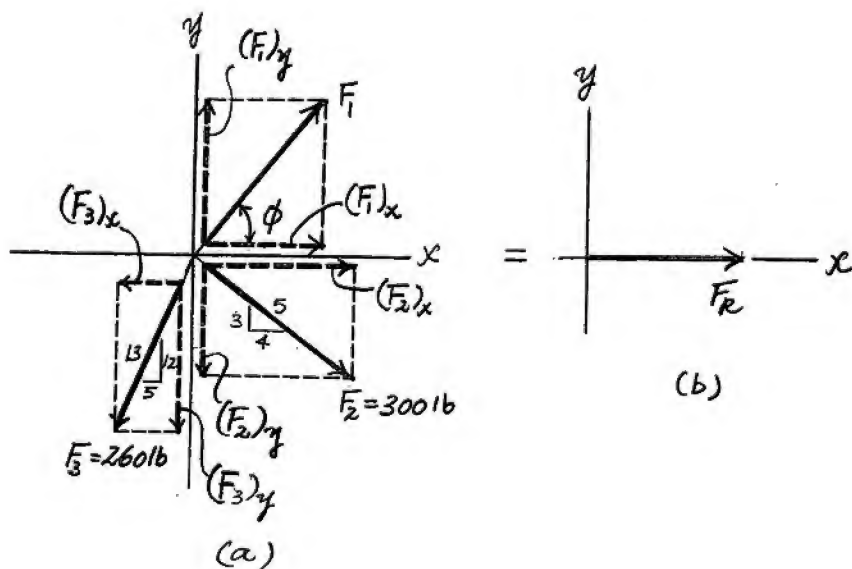
$$+\rightarrow \Sigma (F_R)_x = \Sigma F_x; \quad F_R = F_1 \cos \phi + 240 - 100 \quad (2)$$

By inspecting Eq. (1), we realize that  $F_1$  is minimum when  $\sin \phi = 1$  or  $\phi = 90^\circ$ . Thus,

$$F_1 = 420 \text{ lb} \quad \text{Ans.}$$

Substituting these results into Eq. (2), yields

$$F_R = 140 \text{ lb} \quad \text{Ans.}$$



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2-46. The three concurrent forces acting on the screw eye produce a resultant force  $\mathbf{F}_R = 0$ . If  $F_2 = \frac{2}{3} F_1$  and  $\mathbf{F}_1$  is to be  $90^\circ$  from  $\mathbf{F}_2$  as shown, determine the required magnitude of  $F_3$  expressed in terms of  $F_1$  and the angle  $\theta$ .

**Cartesian Vector Notation :**

$$\begin{aligned}\mathbf{F}_1 &= F_1 \cos 60^\circ \mathbf{i} + F_1 \sin 60^\circ \mathbf{j} \\ &= 0.50F_1 \mathbf{i} + 0.8660F_1 \mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_2 &= \frac{2}{3}F_1 \cos 30^\circ \mathbf{i} - \frac{2}{3}F_1 \sin 30^\circ \mathbf{j} \\ &= 0.5774F_1 \mathbf{i} - 0.3333F_1 \mathbf{j}\end{aligned}$$

$$\mathbf{F}_3 = -F_3 \sin \theta \mathbf{i} - F_3 \cos \theta \mathbf{j}$$

**Resultant Force :**

$$\begin{aligned}\mathbf{F}_R &= \mathbf{0} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ \mathbf{0} &= (0.50F_1 + 0.5774F_1 - F_3 \sin \theta) \mathbf{i} \\ &\quad + (0.8660F_1 - 0.3333F_1 - F_3 \cos \theta) \mathbf{j}\end{aligned}$$

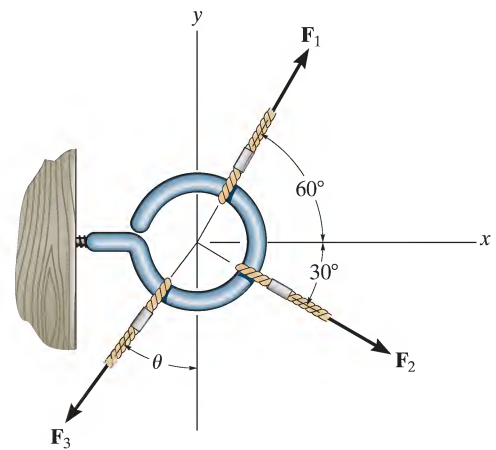
Equating  $\mathbf{i}$  and  $\mathbf{j}$  components, we have

$$0.50F_1 + 0.5774F_1 - F_3 \sin \theta = 0 \quad [1]$$

$$0.8660F_1 - 0.3333F_1 - F_3 \cos \theta = 0 \quad [2]$$

Solving Eq. [1] and [2] yields

$$\theta = 63.7^\circ \quad F_3 = 1.20F_1 \quad \text{Ans}$$



2-47. Determine the magnitude of  $F_A$  and its direction  $\theta$  so that the resultant force is directed along the positive  $x$  axis and has a magnitude of 1250 N.

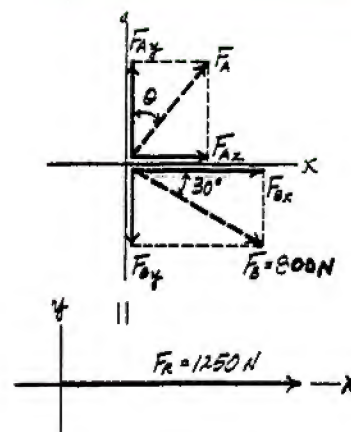
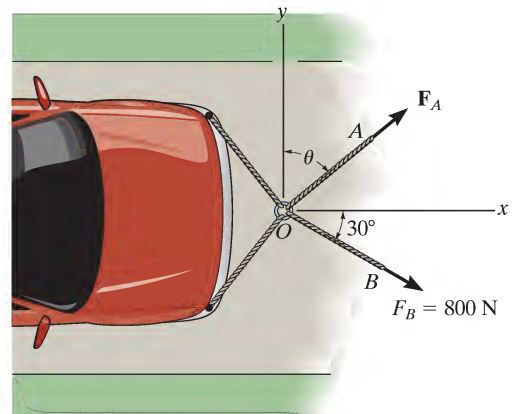
**Scalar Notation :** Summing the force components algebraically, we have

$$\begin{aligned}\rightarrow F_{R_x} &= \Sigma F_x; \quad 1250 = F_A \sin \theta + 800 \cos 30^\circ \\ F_A \sin \theta &= 557.18 \quad [1]\end{aligned}$$

$$\begin{aligned}+\uparrow F_{R_y} &= \Sigma F_y; \quad 0 = F_A \cos \theta - 800 \sin 30^\circ \\ F_A \cos \theta &= 400 \quad [2]\end{aligned}$$

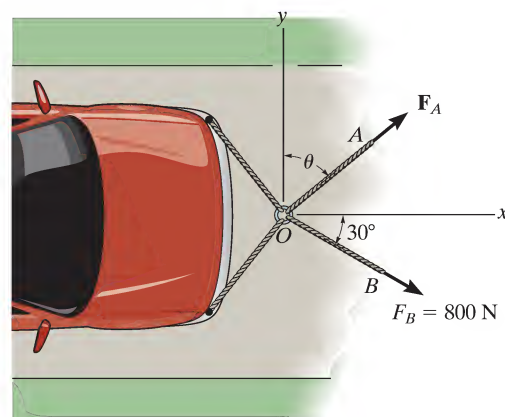
Solving Eq. [1] and [2] yields

$$\theta = 54.3^\circ \quad F_A = 686 \text{ N} \quad \text{Ans}$$



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\*2–48. Determine the magnitude and direction measured counterclockwise from the positive  $x$  axis of the resultant force acting on the ring at  $O$  if  $F_A = 750$  N and  $\theta = 45^\circ$ .



**Scalar Notation :** Summing the force components algebraically, we have

$$\rightarrow F_R = \Sigma F_x; \quad F_{Rx} = 750 \sin 45^\circ + 800 \cos 30^\circ = 1223.15 \text{ N} \rightarrow$$

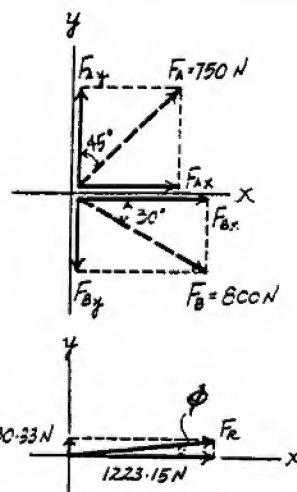
$$+\uparrow F_R = \Sigma F_y; \quad F_{Ry} = 750 \cos 45^\circ - 800 \sin 30^\circ = 130.33 \text{ N} \uparrow$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \quad \text{Ans}$$

The directional angle  $\theta$  measured counterclockwise from positive  $x$  axis is

$$\phi = \tan^{-1} \frac{F_{Ry}}{F_{Rx}} = \tan^{-1} \left( \frac{130.33}{1223.15} \right) = 6.08^\circ \quad \text{Ans}$$



•2–49. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive  $x$  axis.

$$\mathbf{F}_1 = -60 \left( \frac{1}{\sqrt{2}} \right) \mathbf{i} + 60 \left( \frac{1}{\sqrt{2}} \right) \mathbf{j} = \{-42.43 \mathbf{i} + 42.43 \mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_2 = -70 \sin 60^\circ \mathbf{i} - 70 \cos 60^\circ \mathbf{j} = \{-60.62 \mathbf{i} - 35 \mathbf{j}\} \text{ lb}$$

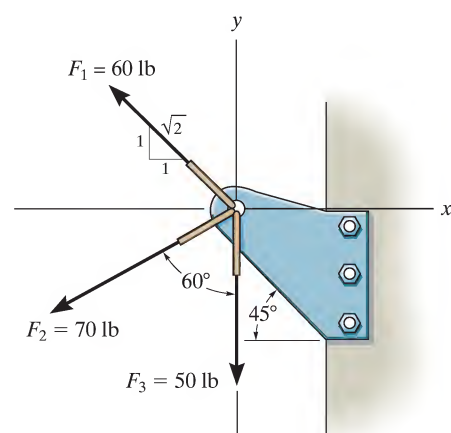
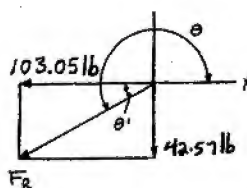
$$\mathbf{F}_3 = \{-50 \mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_R = \Sigma \mathbf{F} = \{-103.05 \mathbf{i} - 42.57 \mathbf{j}\} \text{ lb}$$

$$F_R = \sqrt{(-103.05)^2 + (-42.57)^2} = 111 \text{ lb} \quad \text{Ans}$$

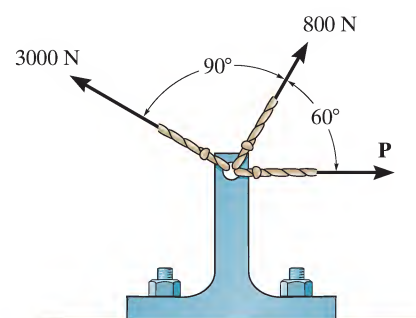
$$\theta' = \tan^{-1} \left( \frac{42.57}{103.05} \right) = 22.4^\circ$$

$$\theta = 180^\circ + 22.4^\circ = 202^\circ \quad \text{Ans}$$



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**2-50.** The three forces are applied to the bracket. Determine the range of values for the magnitude of force **P** so that the resultant of the three forces does not exceed 2400 N.



$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = P + 800 \cos 60^\circ - 3000 \cos 30^\circ = P - 2198.08$$

$$+ \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 800 \sin 60^\circ + 3000 \sin 30^\circ = 2192.82$$

$$F_R = \sqrt{(P - 2198.08)^2 + (2192.82)^2} \leq 2400$$

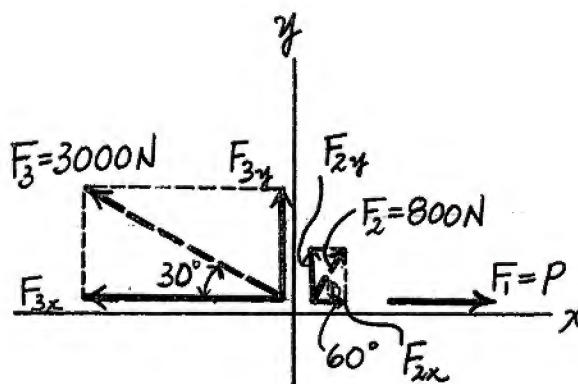
$$(P - 2198.08)^2 + (2192.82)^2 \leq (2400)^2$$

$$|P - 2198.08| \leq 975.47$$

$$-975.47 \leq P - 2198.08 \leq 975.47$$

$$1222.6 \text{ N} \leq P \leq 3173.5 \text{ N}$$

$$1.22 \text{ kN} \leq P \leq 3.17 \text{ kN} \quad \text{Ans}$$



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2-51. If  $F_1 = 150$  N and  $\phi = 30^\circ$ , determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive  $x$  axis.

**Rectangular Components:** By referring to Fig. *a*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ , and  $F_3$  can be written as

$$\begin{aligned} (F_1)_x &= 150 \sin 30^\circ = 75 \text{ N} & (F_1)_y &= 150 \cos 30^\circ = 129.90 \text{ N} \\ (F_2)_x &= 200 \text{ N} & (F_2)_y &= 0 \\ (F_3)_x &= 260 \left( \frac{5}{13} \right) = 100 \text{ N} & (F_3)_y &= 260 \left( \frac{12}{13} \right) = 240 \text{ N} \end{aligned}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

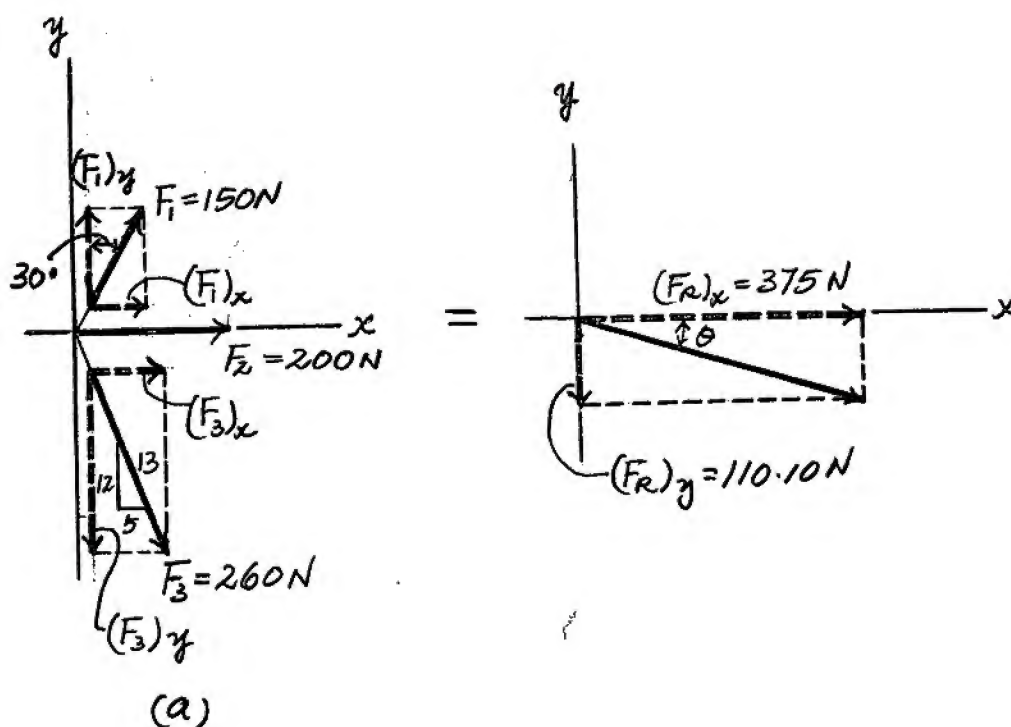
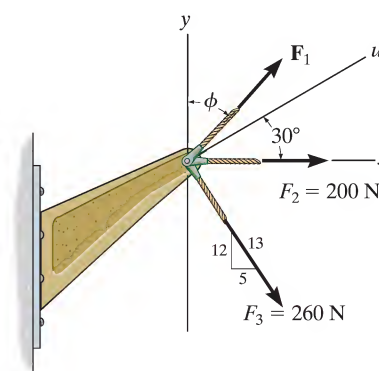
$$\begin{aligned} + \rightarrow \Sigma (F_R)_x &= \Sigma F_x; & (F_R)_x &= 75 + 200 + 100 = 375 \text{ N} \rightarrow \\ + \uparrow \Sigma (F_R)_y &= \Sigma F_y; & (F_R)_y &= 129.90 - 240 = -110.10 \text{ N} = 110.10 \text{ N} \downarrow \end{aligned}$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{375^2 + 110.10^2} = 391 \text{ N} \quad \text{Ans.}$$

The direction angle  $\theta$  of  $F_R$ , Fig. *b*, measured clockwise from the positive  $x$  axis, is

$$\theta = \tan^{-1} \left[ \frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left( \frac{110.10}{375} \right) = 16.4^\circ \quad \text{Ans.}$$



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**\*2-52.** If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive  $u$  axis, determine the magnitude of  $F_1$  and its direction  $\phi$ .

**Rectangular Components:** By referring to Fig.  $a$ , the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_R$  can be written as

$$(F_1)_x = F_1 \sin \phi$$

$$(F_1)_y = F_1 \cos \phi$$

$$(F_2)_x = 200 \text{ N}$$

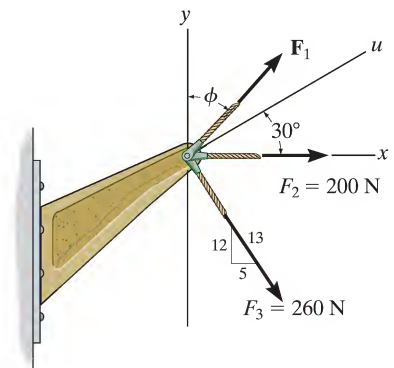
$$(F_2)_y = 0$$

$$(F_3)_x = 260 \left( \frac{5}{13} \right) = 100 \text{ N}$$

$$(F_3)_y = 260 \left( \frac{12}{13} \right) = 240 \text{ lb}$$

$$(F_R)_x = 450 \cos 30^\circ = 389.71 \text{ N}$$

$$(F_R)_y = 450 \sin 30^\circ = 225 \text{ lb}$$



**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned} \rightarrow \Sigma (F_R)_x = \Sigma F_x; \quad 389.71 &= F_1 \sin \phi + 200 + 100 \\ F_1 \sin \phi &= 89.71 \end{aligned} \quad (1)$$

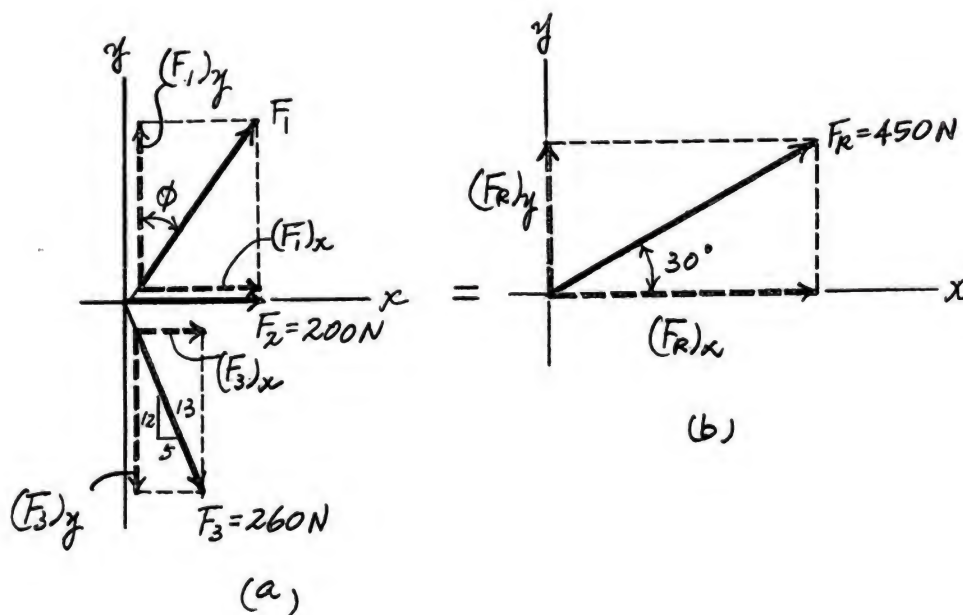
$$\begin{aligned} + \uparrow \Sigma (F_R)_y = \Sigma F_y; \quad 225 &= F_1 \cos \phi - 240 \\ F_1 \cos \phi &= 465 \end{aligned} \quad (2)$$

Solving Eqs. (1) and (2), yields

$$\phi = 10.9^\circ$$

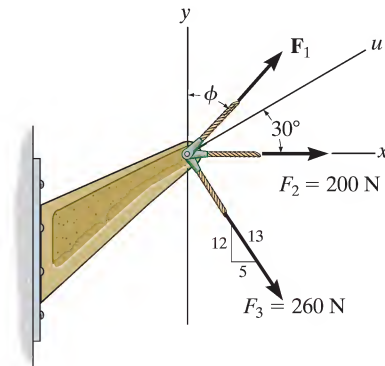
$$F_1 = 474 \text{ N}$$

**Ans.**



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•2–53. If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of  $F_1$  and the resultant force. Set  $\phi = 30^\circ$ .



**Rectangular Components:** By referring to Fig. *a*, the  $x$  and  $y$  components of  $F_1$ ,  $F_2$ , and  $F_3$  can be written as

$$\begin{aligned}(F_1)_x &= F_1 \sin 30^\circ = 0.5F_1 & (F_1)_y &= F_1 \cos 30^\circ = 0.8660F_1 \\ (F_2)_x &= 200 \text{ N} & (F_2)_y &= 0 \\ (F_3)_x &= 260 \left( \frac{5}{13} \right) = 100 \text{ N} & (F_3)_y &= 260 \left( \frac{12}{13} \right) = 240 \text{ N}\end{aligned}$$

**Resultant Force:** Summing the force components algebraically along the  $x$  and  $y$  axes,

$$\begin{aligned}+\rightarrow \Sigma (F_R)_x &= \Sigma F_x; & (F_R)_x &= 0.5F_1 + 200 + 100 \\ & & &= 0.5F_1 + 300 \\ +\uparrow \Sigma (F_R)_y &= \Sigma F_y; & (F_R)_y &= 0.8660F_1 - 240\end{aligned}$$

The magnitude of the resultant force  $F_R$  is

$$\begin{aligned}F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2} \\ &= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2} \\ &= \sqrt{F_1^2 - 115.69F_1 + 147600}\end{aligned}\quad (1)$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147600 \quad (2)$$

The first derivative of Eq. (2) is

$$2F_R \frac{dF_R}{dF_1} = 2F_1 - 115.69 \quad (3)$$

and the second derivative of Eq. (1) is

$$F_R \frac{d^2 F_R}{dF_1^2} + \frac{dF_R}{dF_1} \frac{dF_R}{dF_1} = 1 \quad (4)$$

For  $F_R$  to be minimum,  $\frac{dF_R}{dF_1} = 0$ . Thus, from Eq. (3)

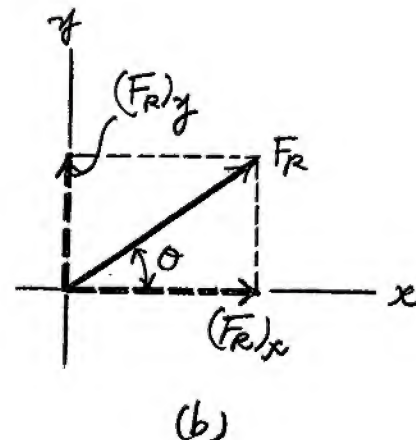
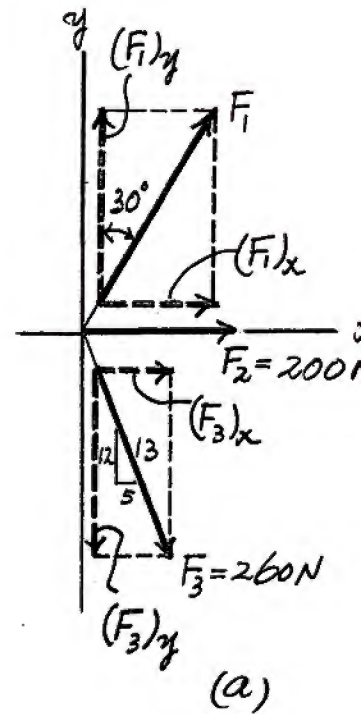
$$\begin{aligned}2F_R \frac{dF_R}{dF_1} &= 2F_1 - 115.69 = 0 \\ F_1 &= 57.84 \text{ N} = 57.8 \text{ N}\end{aligned}$$

Substituting  $F_1 = 57.84 \text{ N}$  and  $\frac{dF_R}{dF_1} = 0$  into Eq. (4),

$$\frac{d^2 F_R}{dF_1^2} = 0.00263 > 0$$

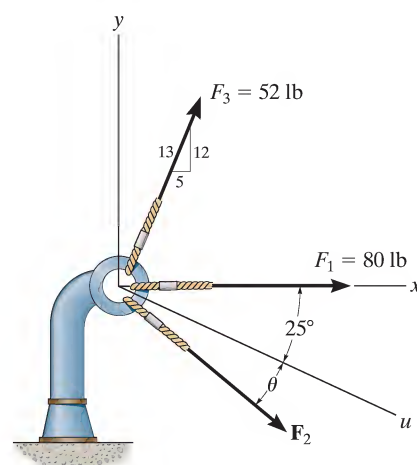
Thus,  $F_1 = 57.84 \text{ N}$  produces a minimum  $F_R$ . From Eq. (1),

$$F_R = \sqrt{(57.84)^2 - 115.69(57.84) + 147600} = 380 \text{ N} \quad \text{Ans.}$$



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**2–54.** Three forces act on the bracket. Determine the magnitude and direction  $\theta$  of  $\mathbf{F}_2$  so that the resultant force is directed along the positive  $u$  axis and has a magnitude of 50 lb.



**Scalar Notation :** Summing the force components algebraically, we have

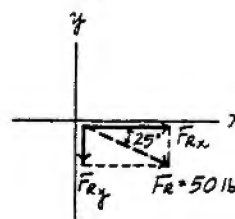
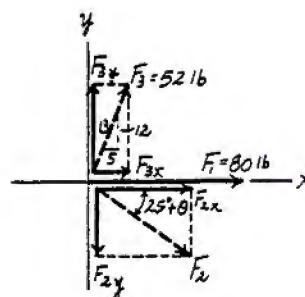
$$\begin{aligned} \rightarrow F_x = \Sigma F_x: \quad 50 \cos 25^\circ &= 80 + 52 \left( \frac{5}{13} \right) + F_2 \cos (25^\circ + \theta) \\ F_2 \cos (25^\circ + \theta) &= -54.684 \end{aligned} \quad [1]$$

$$\begin{aligned} + \uparrow F_y = \Sigma F_y: \quad -50 \sin 25^\circ &= 52 \left( \frac{12}{13} \right) - F_2 \sin (25^\circ + \theta) \\ F_2 \sin (25^\circ + \theta) &= 69.131 \end{aligned} \quad [2]$$

Solving Eq. [1] and [2] yields

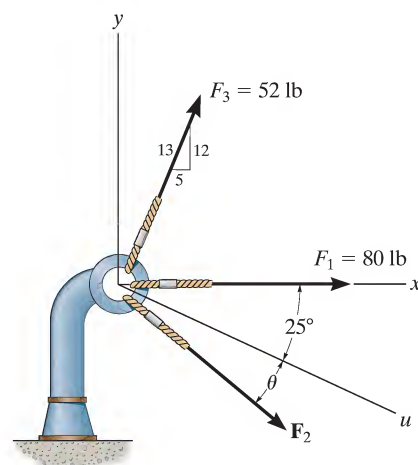
$$25^\circ + \theta = 128.35^\circ \quad \theta = 103^\circ \quad \text{Ans}$$

$$F_2 = 88.1 \text{ lb} \quad \text{Ans}$$



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2-55. If  $F_2 = 150$  lb and  $\theta = 55^\circ$ , determine the magnitude and direction measured clockwise from the positive  $x$  axis of the resultant force of the three forces acting on the bracket.



**Scalar Notation :** Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 80 + 52\left(\frac{5}{13}\right) + 150\cos 80^\circ \\ &= 126.05 \text{ lb} \rightarrow \end{aligned}$$

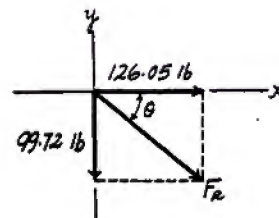
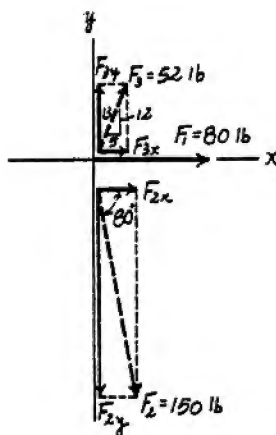
$$\begin{aligned} + \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} &= 52\left(\frac{12}{13}\right) - 150\sin 80^\circ \\ &= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow \end{aligned}$$

The magnitude of the resultant force  $F_R$  is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \quad \text{Ans}$$

The directional angle  $\theta$  measured clockwise from positive  $x$  axis is

$$\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}} = \tan^{-1} \left( \frac{99.72}{126.05} \right) = 38.3^\circ \quad \text{Ans}$$



\*2-56. The three concurrent forces acting on the post produce a resultant force  $F_R = 0$ . If  $F_2 = \frac{1}{2} F_1$ , and  $F_1$  is to be  $90^\circ$  from  $F_2$  as shown, determine the required magnitude of  $F_3$  expressed in terms of  $F_1$  and the angle  $\theta$ .

$$\Sigma F_{x'} = 0; \quad F_3 \cos(\theta - 90^\circ) = F_1$$

$$\Sigma F_{y'} = 0; \quad F_3 \sin(\theta - 90^\circ) = F_2$$

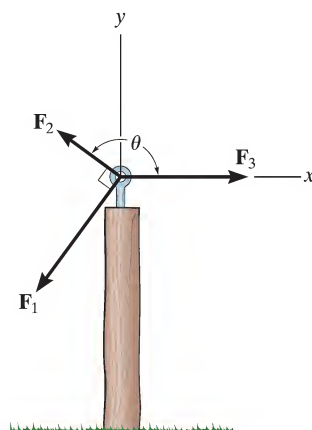
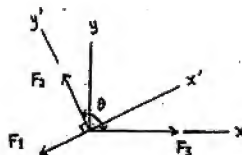
$$\tan(\theta - 90^\circ) = \frac{F_2}{F_1} = \frac{1}{2}$$

$$\theta - 90^\circ = 26.57^\circ$$

$$\theta = 116.57^\circ = 117^\circ \quad \text{Ans}$$

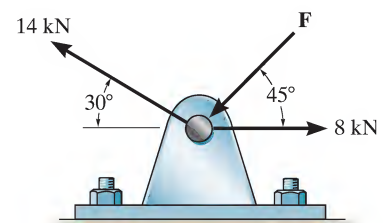
$$F_3 = \frac{F_1}{\cos(116.57^\circ - 90^\circ)}$$

$$F_3 = 1.12 F_1 \quad \text{Ans}$$



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•2-57. Determine the magnitude of force  $F$  so that the resultant force of the three forces is as small as possible. What is the magnitude of this smallest resultant force?



$$\begin{aligned} \rightarrow F_{Rx} = \Sigma F_x: \quad F_{Rx} &= 8 - F \cos 45^\circ - 14 \cos 30^\circ \\ &= -4.1244 - F \cos 45^\circ \end{aligned}$$

$$\begin{aligned} +\uparrow F_{Ry} = \Sigma F_y: \quad F_{Ry} &= -F \sin 45^\circ + 14 \sin 30^\circ \\ &= 7 - F \sin 45^\circ \end{aligned}$$

$$F_R^2 = (-4.1244 - F \cos 45^\circ)^2 + (7 - F \sin 45^\circ)^2 \quad (1)$$

$$2F_R \frac{dF_R}{dF} = 2(-4.1244 - F \cos 45^\circ)(-\cos 45^\circ) + 2(7 - F \sin 45^\circ)(-\sin 45^\circ) = 0$$

$$F = 2.03 \text{ kN} \quad \text{Ans}$$

$$\text{From Eq. (1):} \quad F_R = 7.87 \text{ kN} \quad \text{Ans}$$

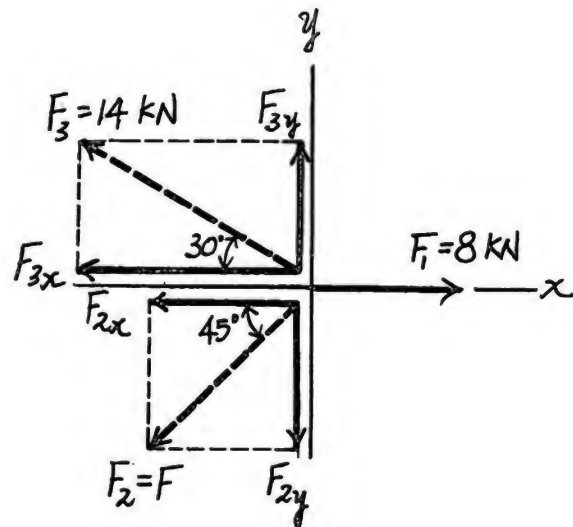
Also, from the figure require

$$(F_R)_x = 0 = \Sigma F_x: \quad F + 14 \sin 15^\circ - 8 \cos 45^\circ = 0$$

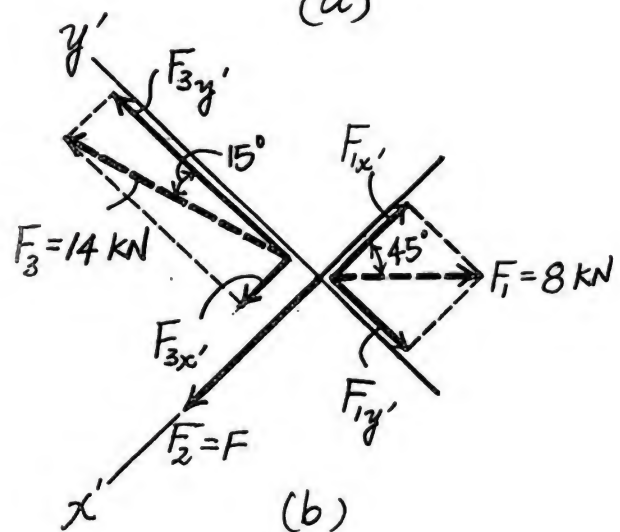
$$F = 2.03 \text{ kN} \quad \text{Ans}$$

$$(F_R)_y = \Sigma F_y: \quad F_R = 14 \cos 15^\circ - 8 \sin 45^\circ$$

$$F_R = 7.87 \text{ kN} \quad \text{Ans}$$



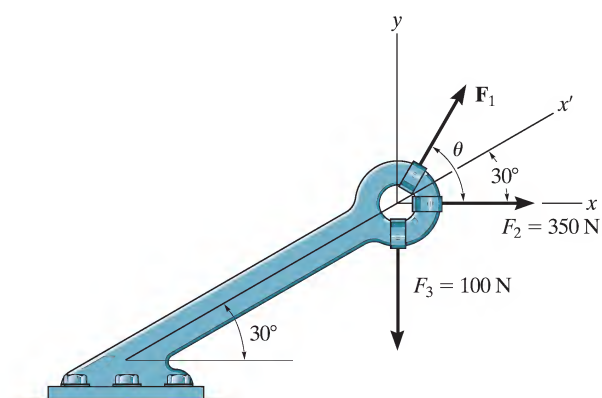
(a)



(b)

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**2-58.** Express each of the three forces acting on the bracket in Cartesian vector form with respect to the  $x$  and  $y$  axes. Determine the magnitude and direction  $\theta$  of  $\mathbf{F}_1$  so that the resultant force is directed along the positive  $x'$  axis and has a magnitude of  $F_R = 600$  N.



$$\mathbf{F}_1 = \{F_1 \cos \theta \mathbf{i} + F_1 \sin \theta \mathbf{j}\} \text{ N} \quad \text{Ans}$$

$$\mathbf{F}_2 = \{350\mathbf{i}\} \text{ N} \quad \text{Ans}$$

$$\mathbf{F}_3 = \{-100\mathbf{j}\} \text{ N} \quad \text{Ans}$$

Require,

$$\mathbf{F}_R = 600 \cos 30^\circ \mathbf{i} + 600 \sin 30^\circ \mathbf{j}$$

$$\mathbf{F}_R = \{519.6\mathbf{i} + 300\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

Equating the  $\mathbf{i}$  and  $\mathbf{j}$  components yields:

$$519.6 = F_1 \cos \theta + 350$$

$$F_1 \cos \theta = 169.6$$

$$300 = F_1 \sin \theta - 100$$

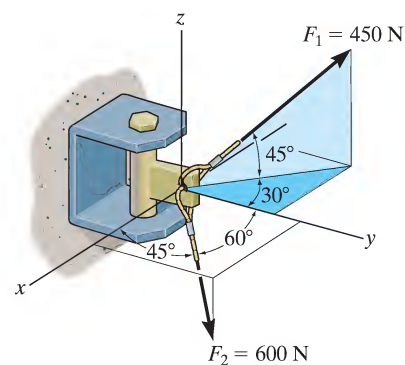
$$F_1 \sin \theta = 400$$

$$\theta = \tan^{-1} \left[ \frac{400}{169.6} \right] = 67.0^\circ \quad \text{Ans}$$

$$F_1 = 434 \text{ N} \quad \text{Ans}$$

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2-59. Determine the coordinate angle  $\gamma$  for  $\mathbf{F}_2$  and then express each force acting on the bracket as a Cartesian vector.



**Rectangular Components:** Since  $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$ , then  $\cos \gamma_2 = \pm \sqrt{1 - \cos^2 45^\circ - \cos^2 60^\circ} = \pm 0.5$ .

However, it is required that  $\gamma_2 > 90^\circ$ , thus,  $\gamma_2 = \cos^{-1}(-0.5) = 120^\circ$ . By resolving  $\mathbf{F}_1$  and  $\mathbf{F}_2$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs. *a* and *b*, respectively  $\mathbf{F}_1$  and  $\mathbf{F}_2$  can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 450 \cos 45^\circ \sin 30^\circ (-\mathbf{i}) + 450 \cos 45^\circ \cos 30^\circ (+\mathbf{j}) + 450 \sin 45^\circ (+\mathbf{k})$$

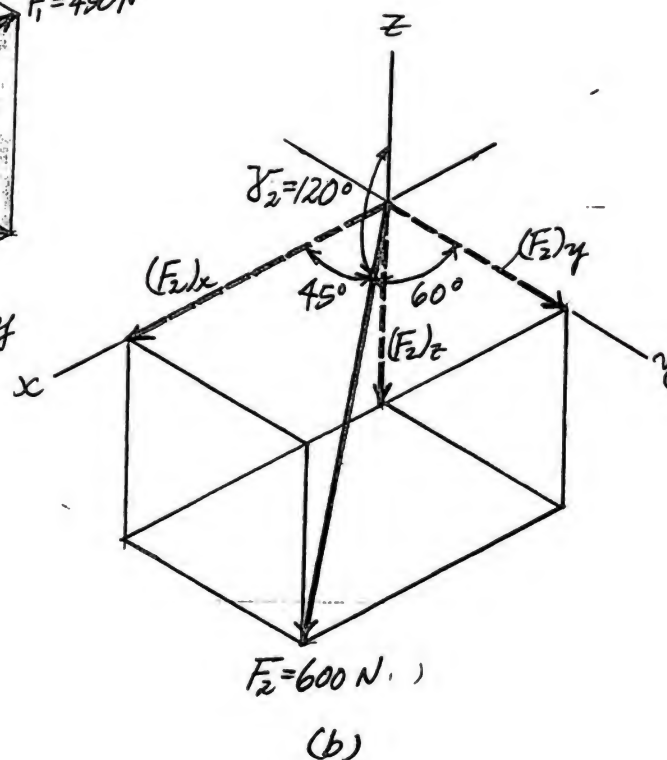
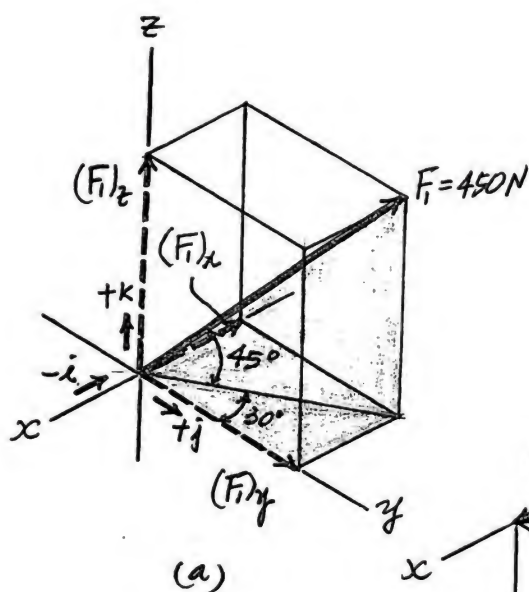
$$= \{-159\mathbf{i} + 276\mathbf{j} + 318\mathbf{k}\} \text{ N}$$

Ans.

$$\mathbf{F}_2 = 600 \cos 45^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 120^\circ \mathbf{k}$$

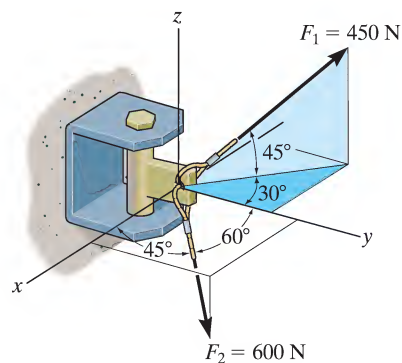
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\} \text{ N}$$

Ans.



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**\*2–60.** Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



**Rectangular Components:** Since  $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$ , then  $\cos \gamma_2 = \pm \sqrt{1 - \cos^2 45^\circ - \cos^2 60^\circ} = \pm 0.5$ .

However, it is required that  $\alpha_2 > 90^\circ$ , thus,  $\gamma_2 = \cos^{-1}(-0.5) = 120^\circ$ . By resolving  $F_1$  and  $F_2$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs. *a* and *b*, respectively,  $F_1$  and  $F_2$ , can be expressed in Cartesian vector form, as

$$F_1 = 450 \cos 45^\circ \sin 30^\circ (-\mathbf{i}) + 450 \cos 45^\circ \cos 30^\circ (+\mathbf{j}) + 450 \sin 45^\circ (+\mathbf{k})$$

$$= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\} \text{ N}$$

**Ans.**

$$F_2 = 600 \cos 45^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 120^\circ \mathbf{k}$$

$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\} \text{ N}$$

**Ans.**

**Resultant Force:** By adding  $F_1$  and  $F_2$  vectorally, we obtain  $F_R$ .

$$F_R = F_1 + F_2$$

$$= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})$$

$$= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\} \text{ N}$$

The magnitude of  $F_R$  is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97 \text{ N} = 634 \text{ N}$$

**Ans.**

The coordinate direction angles of  $F_R$  are

$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{265.16}{633.97} \right) = 65.3^\circ$$

**Ans.**

$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{575.57}{633.97} \right) = 24.8^\circ$$

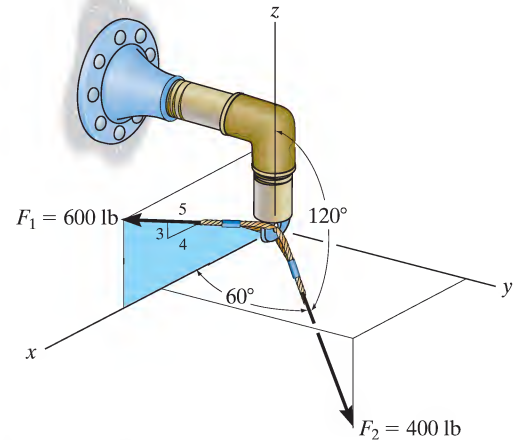
**Ans.**

$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{18.20}{633.97} \right) = 88.4^\circ$$

**Ans.**

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•2–61. Express each force acting on the pipe assembly in Cartesian vector form.



**Rectangular Components:** Since  $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$ , then  $\cos \beta_2 = \pm \sqrt{1 - \cos^2 60^\circ - \cos^2 120^\circ} = \pm 0.7071$ .

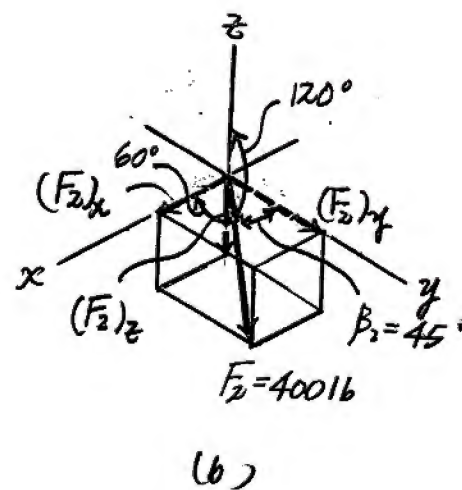
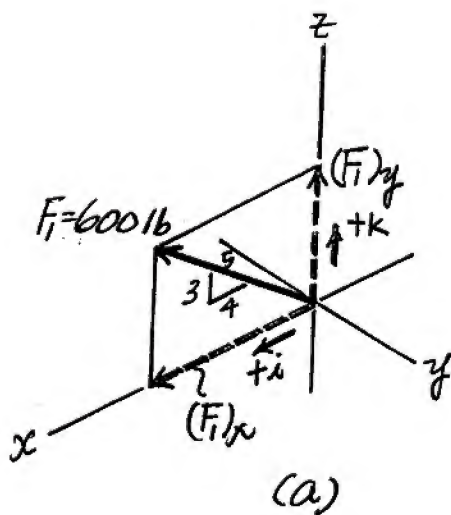
However, it is required that  $\beta_2 > 90^\circ$ , thus,  $\beta_2 = \cos^{-1}(0.7071) = 45^\circ$ . By resolving  $F_1$  and  $F_2$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs. *a* and *b*, respectively,  $F_1$  and  $F_2$ , can be expressed in Cartesian vector form, as

$$\begin{aligned} F_1 &= 600 \left( \frac{4}{5} \right) (+i) + 0j + 600 \left( \frac{3}{5} \right) (+k) \\ &= [480i + 360k] \text{ N} \end{aligned}$$

Ans.

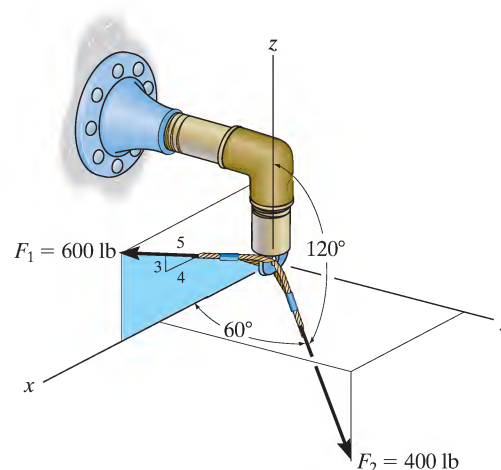
$$\begin{aligned} F_2 &= 400 \cos 60^\circ i + 400 \cos 45^\circ j + 400 \cos 120^\circ k \\ &= [200i + 283j - 200k] \text{ N} \end{aligned}$$

Ans.



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**2-62.** Determine the magnitude and direction of the resultant force acting on the pipe assembly.



**Force Vectors:** Since  $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$ , then  $\cos \gamma_2 = \pm \sqrt{1 - \cos^2 60^\circ - \cos^2 120^\circ} = \pm 0.7071$ .

However, it is required that  $\gamma_2 < 90^\circ$ , thus,  $\gamma_2 = \cos^{-1}(0.7071) = 45^\circ$ . By resolving  $F_1$  and  $F_2$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs. *a* and *b*, respectively,  $F_1$  and  $F_2$ , can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600 \left( \frac{4}{5} \right) (+\mathbf{i}) + 0\mathbf{j} + 600 \left( \frac{3}{5} \right) (+\mathbf{k})$$

$$= \{480\mathbf{i} + 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = 400 \cos 60^\circ \mathbf{i} + 400 \cos 45^\circ \mathbf{j} + 400 \cos 120^\circ \mathbf{k}$$

$$= \{200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k}\} \text{ lb}$$

**Resultant Force:** By adding  $F_1$  and  $F_2$  vectorally, we obtain  $F_R$ .

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (480\mathbf{i} + 360\mathbf{k}) + (200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k})$$

$$= \{680\mathbf{i} + 282.84\mathbf{j} + 160\mathbf{k}\} \text{ lb}$$

The magnitude of  $F_R$  is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{680^2 + 282.84^2 + 160^2} = 753.66 \text{ lb} = 754 \text{ lb} \quad \text{Ans.}$$

The coordinate direction angles of  $F_R$  are

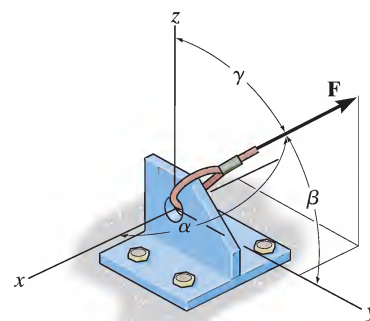
$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{680}{753.66} \right) = 25.5^\circ \quad \text{Ans.}$$

$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{282.84}{753.66} \right) = 68.0^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{160}{753.66} \right) = 77.7^\circ \quad \text{Ans.}$$

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**2-63.** The force  $\mathbf{F}$  acts on the bracket within the octant shown. If  $F = 400\text{ N}$ ,  $\beta = 60^\circ$ , and  $\gamma = 45^\circ$ , determine the  $x, y, z$  components of  $\mathbf{F}$ .



**Coordinate Direction Angles:** Since  $\beta$  and  $\gamma$  are known, the third angle  $\alpha$  can be determined from

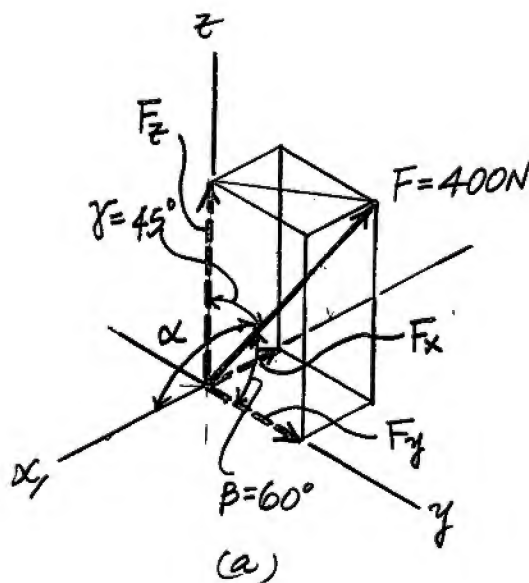
$$\begin{aligned}\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= 1 \\ \cos^2 \alpha + \cos^2 60^\circ + \cos^2 45^\circ &= 1 \\ \cos \alpha &= \pm 0.5\end{aligned}$$

Since  $\mathbf{F}$  is in the octant shown in Fig.  $a$ ,  $\theta_x$  must be greater than  $90^\circ$ . Thus,  
 $\alpha = \cos^{-1}(-0.5) = 120^\circ$ .

**Rectangular Components:** By referring to Fig.  $a$ , the  $x, y$ , and  $z$  components of  $\mathbf{F}$  can be written as

$$\begin{aligned}F_x &= F \cos \alpha = 400 \cos 120^\circ = -200\text{ N} && \text{Ans.} \\ F_y &= F \cos \beta = 400 \cos 60^\circ = 200\text{ N} && \text{Ans.} \\ F_z &= F \cos \gamma = 400 \cos 45^\circ = 283\text{ N} && \text{Ans.}\end{aligned}$$

The negative sign indicates that  $F_x$  is directed towards the negative  $x$ -axis.



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\*2-64. The force  $\mathbf{F}$  acts on the bracket within the octant shown. If the magnitudes of the  $x$  and  $z$  components of  $\mathbf{F}$  are  $F_x = 300\text{ N}$  and  $F_z = 600\text{ N}$ , respectively, and  $\beta = 60^\circ$ , determine the magnitude of  $\mathbf{F}$  and its  $y$  component. Also, find the coordinate direction angles  $\alpha$  and  $\gamma$ .

**Rectangular Components:** The magnitude of  $\mathbf{F}$  is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{300^2 + F_y^2 + 600^2}$$

$$F^2 = F_y^2 + 450\,000 \quad (1)$$

The magnitude of  $F_y$  is given by

$$F_y = F \cos 60^\circ = 0.5 F \quad (2)$$

Solving Eqs. (1) and (2) yields

$$F = 774.60\text{ N} \approx 775\text{ N} \quad \text{Ans.}$$

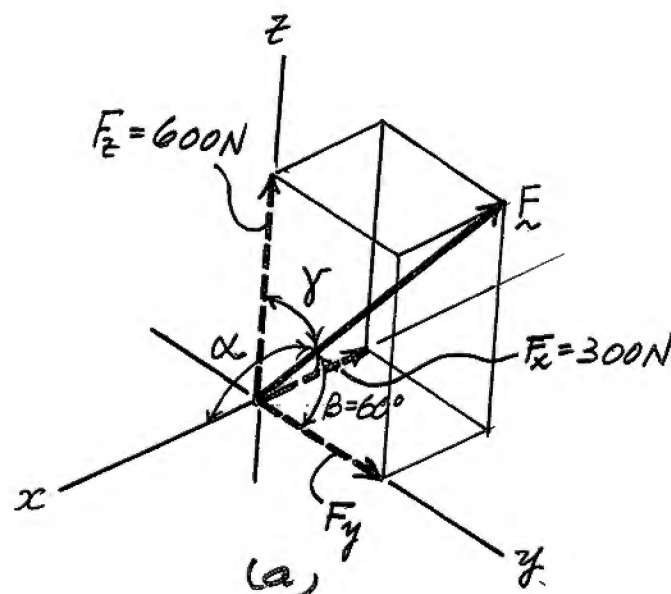
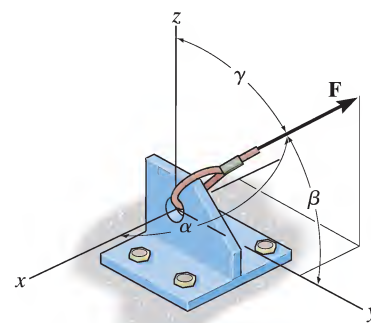
$$F_y = 387\text{ N} \quad \text{Ans.}$$

**Coordinate Direction Angles:** Since  $\mathbf{F}$  is contained in the octant so that  $F_x$  is directed towards the negative  $x$  axis, the coordinate direction angle  $\theta_x$  is given by

$$\alpha = \cos^{-1}\left(\frac{-F_x}{F}\right) = \cos^{-1}\left(\frac{-300}{774.60}\right) = 113^\circ \quad \text{Ans.}$$

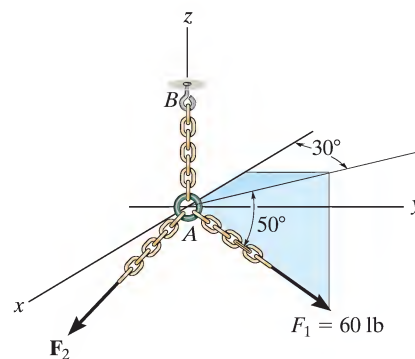
The third coordinate direction angle is

$$\gamma = \cos^{-1}\left(\frac{-F_z}{F}\right) = \cos^{-1}\left(\frac{600}{774.60}\right) = 39.2^\circ \quad \text{Ans.}$$



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•2–65. The two forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  acting at  $A$  have a resultant force of  $\mathbf{F}_R = \{-100\mathbf{k}\}$  lb. Determine the magnitude and coordinate direction angles of  $\mathbf{F}_2$ .



**Cartesian Vector Notation :**

$$\mathbf{F}_R = \{-100\mathbf{k}\} \text{ lb}$$

$$\begin{aligned}\mathbf{F}_1 &= 60\{-\cos 50^\circ \cos 30^\circ \mathbf{i} + \cos 50^\circ \sin 30^\circ \mathbf{j} - \sin 50^\circ \mathbf{k}\} \text{ lb} \\ &= \{-33.40\mathbf{i} + 19.28\mathbf{j} - 45.96\mathbf{k}\} \text{ lb}\end{aligned}$$

$$\mathbf{F}_2 = \{F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\} \text{ lb}$$

**Resultant Force :**

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ -100\mathbf{k} &= \{ (F_{2x} - 33.40)\mathbf{i} + (F_{2y} + 19.28)\mathbf{j} + (F_{2z} - 45.96)\mathbf{k} \}\end{aligned}$$

Equating i, j and k components, we have

$$\begin{aligned}F_{2x} - 33.40 &= 0 & F_{2x} &= 33.40 \text{ lb} \\ F_{2y} + 19.28 &= 0 & F_{2y} &= -19.28 \text{ lb} \\ F_{2z} - 45.96 &= -100 & F_{2z} &= -54.04 \text{ lb}\end{aligned}$$

The magnitude of force  $\mathbf{F}_2$  is

$$\begin{aligned}F_2 &= \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} \\ &= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2} \\ &= 66.39 \text{ lb} = 66.4 \text{ lb}\end{aligned}$$

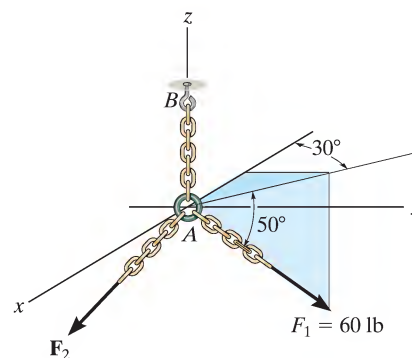
**Ans**

The coordinate direction angles for  $\mathbf{F}_2$  are

$$\begin{aligned}\cos \alpha &= \frac{F_{2x}}{F_2} = \frac{33.40}{66.39} & \alpha &= 59.8^\circ & \text{Ans} \\ \cos \beta &= \frac{F_{2y}}{F_2} = \frac{-19.28}{66.39} & \beta &= 107^\circ & \text{Ans} \\ \cos \gamma &= \frac{F_{2z}}{F_2} = \frac{-54.04}{66.39} & \gamma &= 144^\circ & \text{Ans}\end{aligned}$$

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2-66. Determine the coordinate direction angles of the force  $\mathbf{F}_1$  and indicate them on the figure.

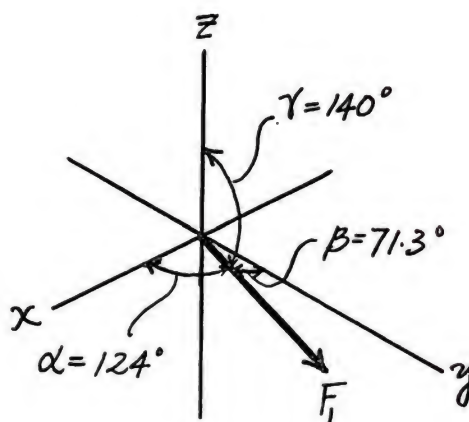


Unit Vector For Force  $\mathbf{F}_1$  :

$$\begin{aligned} \mathbf{u}_{F_1} &= -\cos 50^\circ \cos 30^\circ \mathbf{i} + \cos 50^\circ \sin 30^\circ \mathbf{j} - \sin 50^\circ \mathbf{k} \\ &= -0.5567 \mathbf{i} + 0.3214 \mathbf{j} - 0.7660 \mathbf{k} \end{aligned}$$

Coordinate Direction Angles : From the unit vector obtained above, we have

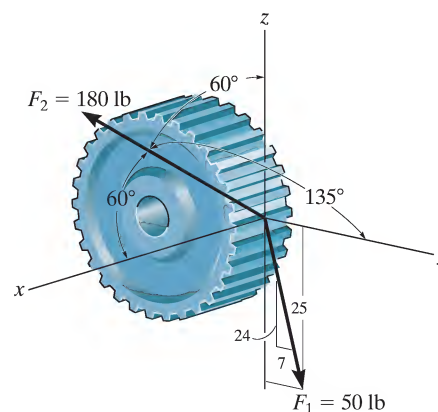
$\cos \alpha = -0.5567$	$\alpha = 124^\circ$	Ans
$\cos \beta = 0.3214$	$\beta = 71.3^\circ$	Ans
$\cos \gamma = -0.7660$	$\gamma = 140^\circ$	Ans



2-67. The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\} \text{ lb} \quad \text{Ans}$$

$$\begin{aligned} \mathbf{F}_2 &= 180 \cos 60^\circ \mathbf{i} + 180 \cos 135^\circ \mathbf{j} + 180 \cos 60^\circ \mathbf{k} \\ &= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\} \text{ lb} \quad \text{Ans} \end{aligned}$$



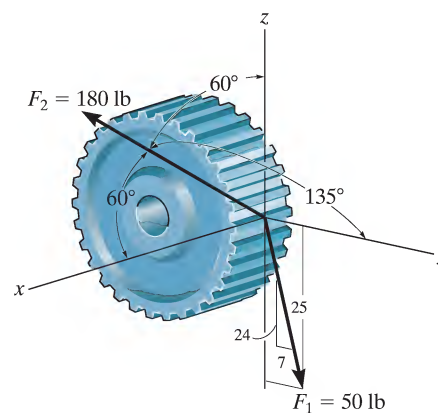
\*2-68. The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.

$$F_{Rx} = 180 \cos 60^\circ = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135^\circ = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60^\circ = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb} \quad \text{Ans}$$



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•2–69. If the resultant force acting on the bracket is  $\mathbf{F}_R = \{-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k}\}$  N, determine the magnitude and coordinate direction angles of  $\mathbf{F}$ .

**Force Vectors:** By resolving  $\mathbf{F}_1$  and  $\mathbf{F}_2$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Fig. *a*.

$\mathbf{F}_1$  and  $\mathbf{F}_2$  can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750 \cos 45^\circ \cos 30^\circ (+\mathbf{i}) + 750 \cos 45^\circ \sin 30^\circ (+\mathbf{j}) + 750 \sin 45^\circ (-\mathbf{k})$$

$$= [459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}] \text{ N}$$

$$\mathbf{F} = F \cos \alpha \mathbf{i} + F \cos \beta \mathbf{j} + F \cos \gamma \mathbf{k}$$

**Resultant Force:** By adding  $\mathbf{F}_1$  and  $\mathbf{F}$  vectorally, we obtain  $\mathbf{F}_R$ . Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F \cos \theta_x \mathbf{i} + F \cos \theta_y \mathbf{j} + F \cos \theta_z \mathbf{k})$$

$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28 + F \cos \theta_x)\mathbf{i} + (265.17 + F \cos \theta_y)\mathbf{j} + (F \cos \theta_z - 530.33)\mathbf{k}$$

Equating the  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  components,

$$-300 = 459.28 + F \cos \alpha$$

$$F \cos \alpha = -759.28$$

(1)

$$650 = 265.17 + F \cos \beta$$

$$F \cos \beta = 384.83$$

(2)

$$250 = F \cos \gamma - 530.33$$

$$F \cos \gamma = 780.33$$

(3)

Squaring and then adding Eqs. (1), (2), and (3), yields

$$F^2(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 1\,333\,518.08$$

(4)

However,  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ . Thus, from Eq. (4)

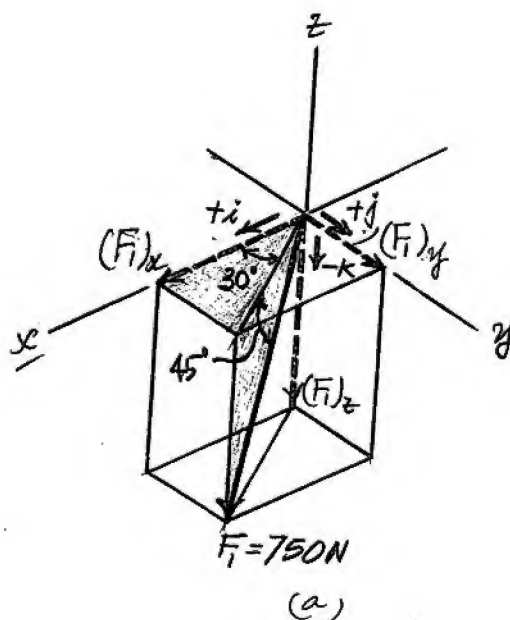
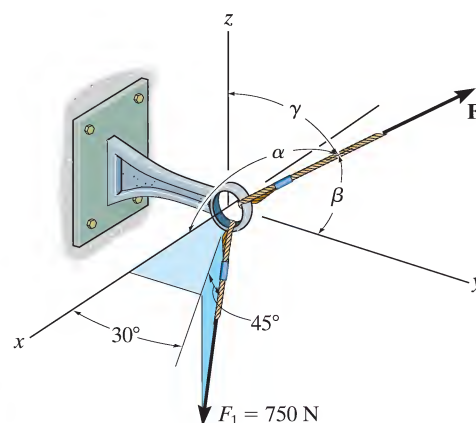
$$F = 1154.78 \text{ N} = 1.15 \text{ kN}$$

Ans.

Substituting  $F = 1154.78 \text{ N}$  into Eqs. (1), (2), and (3), yields

$$\alpha = 131^\circ \quad \beta = 70.5^\circ \quad \gamma = 47.5^\circ$$

Ans.



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2-70. If the resultant force acting on the bracket is to be  $\mathbf{F}_R = \{800\mathbf{j}\}$  N, determine the magnitude and coordinate direction angles of  $\mathbf{F}$ .

**Force Vectors:** By resolving  $\mathbf{F}_1$  and  $\mathbf{F}$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs. *b* and *c*, respectively,  $\mathbf{F}_1$  and  $\mathbf{F}$  can be expressed in Cartesian vector form as

$$\begin{aligned}\mathbf{F}_1 &= 750 \cos 45^\circ \cos 30^\circ (\mathbf{i}) + 750 \cos 45^\circ \sin 30^\circ (\mathbf{j}) + 750 \sin 45^\circ (-\mathbf{k}) \\ &= \{459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}\} \text{ N} \\ \mathbf{F} &= F \cos \alpha \mathbf{i} + F \cos \beta \mathbf{j} + F \cos \gamma \mathbf{k}\end{aligned}$$

**Resultant Force:** By adding  $\mathbf{F}_1$  and  $\mathbf{F}$  vectorally, Figs. *a*, *b*, and *c*, we obtain  $\mathbf{F}_R$ . Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F} \\ 800\mathbf{j} &= (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F \cos \alpha \mathbf{i} + F \cos \beta \mathbf{j} + F \cos \gamma \mathbf{k}) \\ 800\mathbf{j} &= (459.28 + F \cos \alpha)\mathbf{i} + (265.17 + F \cos \beta)\mathbf{j} + (F \cos \gamma - 530.33)\mathbf{k}\end{aligned}$$

Equating the  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  components, we have

$$\begin{aligned}0 &= 459.28 + F \cos \alpha \\ F \cos \alpha &= -459.28\end{aligned}\quad (1)$$

$$\begin{aligned}800 &= 265.17 + F \cos \beta \\ F \cos \beta &= 534.8\end{aligned}\quad (2)$$

$$\begin{aligned}0 &= F \cos \gamma - 530.33 \\ F \cos \gamma &= 530.33\end{aligned}\quad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

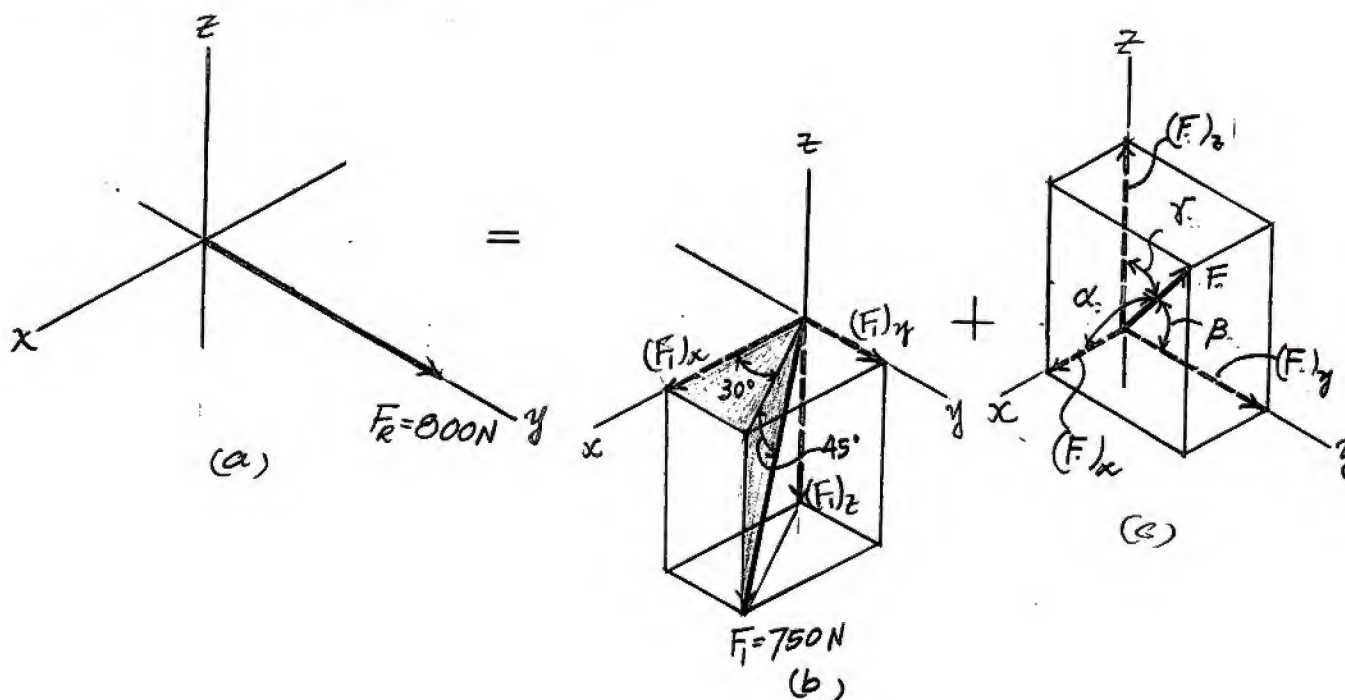
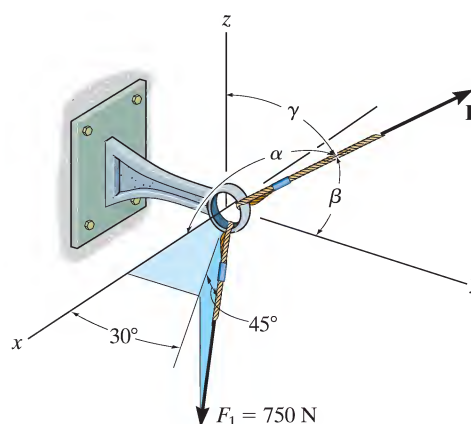
$$F^2(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 778\,235.93\quad (4)$$

However,  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ . Thus, from Eq. (4)

$$F = 882.17 \text{ N} = 882 \text{ N} \quad \text{Ans.}$$

Substituting  $F = 882.17 \text{ N}$  into Eqs. (1), (2), and (3), yields

$$\alpha = 121^\circ \quad \beta = 52.7^\circ \quad \gamma = 53.0^\circ \quad \text{Ans.}$$



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2-71. If  $\alpha = 120^\circ$ ,  $\beta < 90^\circ$ ,  $\gamma = 60^\circ$ , and  $F = 400$  lb, determine the magnitude and coordinate direction angles of the resultant force acting on the hook.

**Force Vectors:** Since  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ , then  $\cos \beta = \pm \sqrt{1 - \cos^2 120^\circ - \cos^2 60^\circ} = \pm 0.7071$ .

However, it is required that  $\beta < 90^\circ$ , thus,  $\beta = \cos^{-1}(0.7071) = 45^\circ$ . By resolving  $F_1$  and  $F_2$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs. *a* and *b*, respectively,  $F_1$  and  $F_2$ , can be expressed in Cartesian vector form as

$$\begin{aligned} F_1 &= 600 \left( \frac{4}{5} \right) \sin 30^\circ (+i) + 600 \left( \frac{4}{5} \right) \cos 30^\circ (+j) + 600 \left( \frac{3}{5} \right) (-k) \\ &= \{240i + 415.69j - 360k\} \text{ lb} \\ F &= 400 \cos 120^\circ i + 400 \cos 45^\circ j + 400 \cos 60^\circ k \\ &= \{-200i + 282.84j + 200k\} \text{ lb} \end{aligned}$$

**Resultant Force:** By adding  $F_1$  and  $F$  vectorially, we obtain  $F_R$ .

$$\begin{aligned} F_R &= F_1 + F \\ &= (240i + 415.69j - 360k) + (-200i + 282.84j + 200k) \\ &= \{40i + 698.53j - 160k\} \text{ lb} \end{aligned}$$

The magnitude of  $F_R$  is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(40)^2 + (698.53)^2 + (-160)^2} = 717.74 \text{ lb} = 718 \text{ lb} \end{aligned}$$

Ans.

The coordinate direction angles of  $F_R$  are

$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{40}{717.74} \right) = 86.8^\circ$$

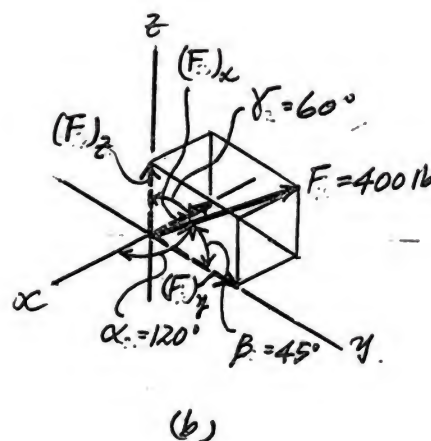
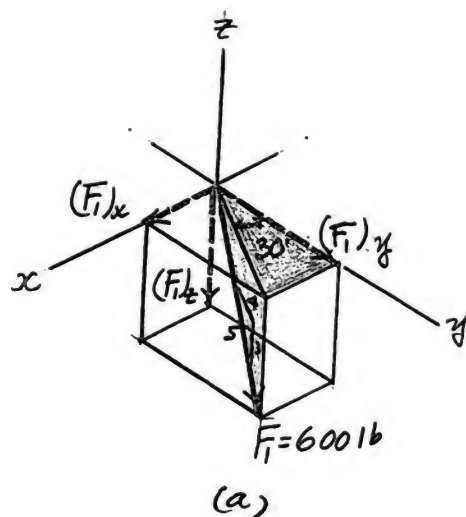
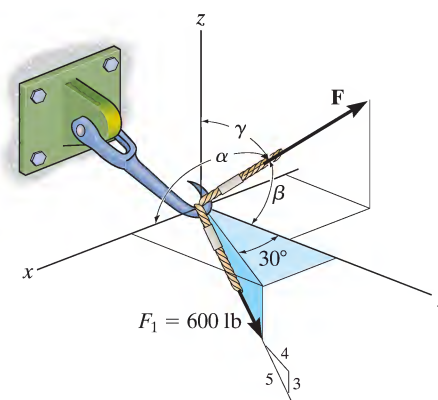
Ans.

$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{698.53}{717.74} \right) = 13.3^\circ$$

Ans.

$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{-160}{717.74} \right) = 103^\circ$$

Ans.



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\*2-72. If the resultant force acting on the hook is  $\mathbf{F}_R = \{-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k}\}$  lb, determine the magnitude and coordinate direction angles of  $\mathbf{F}$ .

**Force Vectors:** By resolving  $\mathbf{F}_1$  and  $\mathbf{F}$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs.  $a$  and  $b$ , respectively,  $\mathbf{F}_1$  and  $\mathbf{F}_2$  can be expressed in Cartesian vector form as

$$\begin{aligned}\mathbf{F}_1 &= 600\left(\frac{4}{5}\right)\sin 30^\circ(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30^\circ(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k}) \\ &= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\} \text{ lb} \\ \mathbf{F} &= F \cos \alpha \mathbf{i} + F \cos \beta \mathbf{j} + F \cos \gamma \mathbf{k}\end{aligned}$$

**Resultant Force:** By adding  $\mathbf{F}_1$  and  $\mathbf{F}_2$  vectorally, we obtain  $\mathbf{F}_R$ . Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F} \\ -200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} &= (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (F \cos \theta_x \mathbf{i} + F \cos \theta_y \mathbf{j} + F \cos \theta_z \mathbf{k}) \\ -200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} &= (240 + F \cos \alpha)\mathbf{i} + (415.69 + F \cos \beta)\mathbf{j} + (F \cos \gamma - 360)\mathbf{k}\end{aligned}$$

Equating the  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  components, we have

$$\begin{aligned}-200 &= 240 + F \cos \alpha \\ F \cos \alpha &= -440\end{aligned}\quad (1)$$

$$\begin{aligned}800 &= 415.69 + F \cos \beta \\ F \cos \beta &= 384.31\end{aligned}\quad (2)$$

$$\begin{aligned}150 &= F \cos \gamma - 360 \\ F \cos \gamma &= 510\end{aligned}\quad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

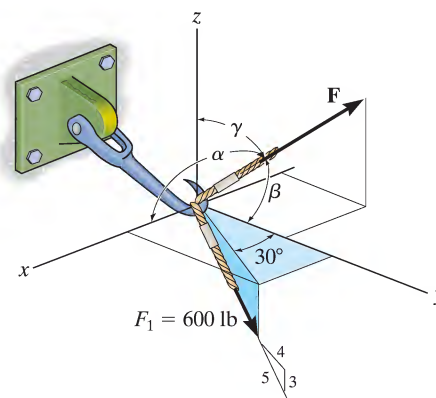
$$F^2(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 601\,392.49\quad (4)$$

However,  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ . Thus, from Eq. (4)

$$F = 775.49 \text{ N} \approx 775 \text{ N} \quad \text{Ans.}$$

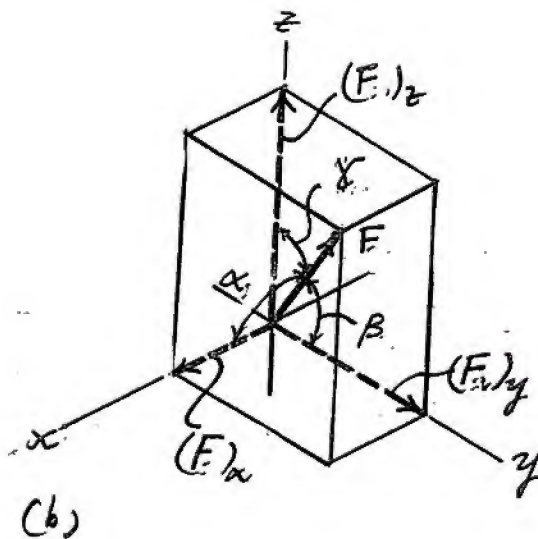
Substituting  $F = 775.49 \text{ N}$  into Eqs. (1), (2), and (3), yields

$$\alpha = 125^\circ \quad \beta = 60.3^\circ \quad \gamma = 48.9^\circ \quad \text{Ans.}$$



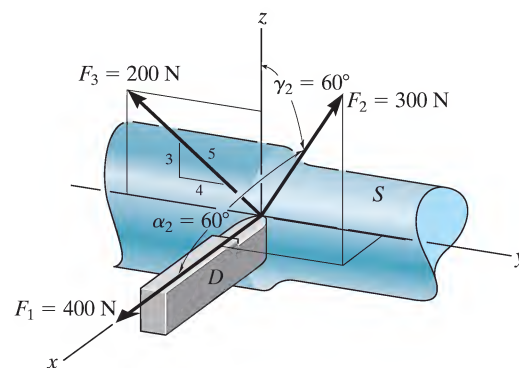
see  
prob.  
2-71a

(a)



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•2–73. The shaft  $S$  exerts three force components on the die  $D$ . Find the magnitude and coordinate direction angles of the resultant force. Force  $\mathbf{F}_2$  acts within the octant shown.



$$\mathbf{F}_1 = 400 \mathbf{i}$$

$$\text{Since } \cos^2 60^\circ + \cos^2 \beta_2 + \cos^2 60^\circ = 1$$

$$\text{Solving for the positive root, } \beta_2 = 45^\circ$$

$$\begin{aligned} \mathbf{F}_2 &= 300 \cos 60^\circ \mathbf{i} + 300 \cos 45^\circ \mathbf{j} + 300 \cos 60^\circ \mathbf{k} \\ &= 150 \mathbf{i} + 212.1 \mathbf{j} + 150 \mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_3 &= -200 \left( \frac{4}{5} \right) \mathbf{j} + 200 \left( \frac{3}{5} \right) \mathbf{k} \\ &= -160 \mathbf{j} + 120 \mathbf{k} \end{aligned}$$

Then

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 550 \mathbf{i} + 52.1 \mathbf{j} + 270 \mathbf{k}$$

$$F_R = \sqrt{(550)^2 + (52.1)^2 + (270)^2} = 614.9 \text{ N} = 615 \text{ N} \quad \text{Ans}$$

$$\alpha = \cos^{-1} \left( \frac{550}{614.9} \right) = 26.6^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1} \left( \frac{52.1}{614.9} \right) = 85.1^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1} \left( \frac{270}{614.9} \right) = 64.0^\circ \quad \text{Ans}$$

2–74. The mast is subjected to the three forces shown. Determine the coordinate direction angles  $\alpha_1, \beta_1, \gamma_1$  of  $\mathbf{F}_1$  so that the resultant force acting on the mast is  $\mathbf{F}_R = \{350\mathbf{i}\} \text{ N}$ .

$$\mathbf{F}_1 = 500 \cos \alpha_1 \mathbf{i} + 500 \cos \beta_1 \mathbf{j} + 500 \cos \gamma_1 \mathbf{k}$$

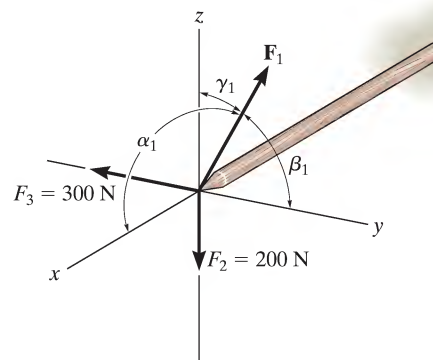
$$\mathbf{F}_R = \mathbf{F}_1 + (-300\mathbf{j}) + (-200\mathbf{k})$$

$$350\mathbf{i} = 500 \cos \alpha_1 \mathbf{i} + (500 \cos \beta_1 - 300)\mathbf{j} + (500 \cos \gamma_1 - 200)\mathbf{k}$$

$$350 = 500 \cos \alpha_1; \quad \alpha_1 = 45.6^\circ \quad \text{Ans}$$

$$0 = 500 \cos \beta_1 - 300; \quad \beta_1 = 53.1^\circ \quad \text{Ans}$$

$$0 = 500 \cos \gamma_1 - 200; \quad \gamma_1 = 66.4^\circ \quad \text{Ans}$$



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2-75. The mast is subjected to the three forces shown. Determine the coordinate direction angles  $\alpha_1, \beta_1, \gamma_1$  of  $\mathbf{F}_1$  so that the resultant force acting on the mast is zero.

$$\mathbf{F}_1 = \{500 \cos \alpha_1 \mathbf{i} + 500 \cos \beta_1 \mathbf{j} + 500 \cos \gamma_1 \mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = \{-200\mathbf{k}\} \text{ N}$$

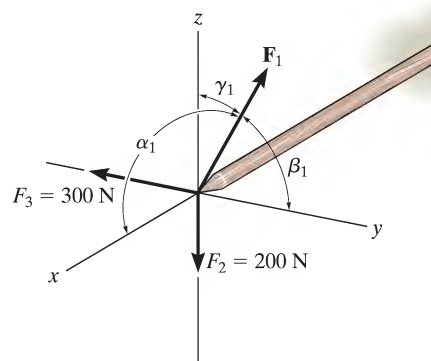
$$\mathbf{F}_3 = \{-300\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

$$500 \cos \alpha_1 = 0; \quad \alpha_1 = 90^\circ \quad \text{Ans}$$

$$500 \cos \beta_1 = 300; \quad \beta_1 = 53.1^\circ \quad \text{Ans}$$

$$500 \cos \gamma_1 = 200; \quad \gamma_1 = 66.4^\circ \quad \text{Ans}$$



\*2-76. Determine the magnitude and coordinate direction angles of  $\mathbf{F}_2$  so that the resultant of the two forces acts along the positive  $x$  axis and has a magnitude of 500 N.

$$\begin{aligned} \mathbf{F}_1 &= (180 \cos 15^\circ) \sin 60^\circ \mathbf{i} + (180 \cos 15^\circ) \cos 60^\circ \mathbf{j} - 180 \sin 15^\circ \mathbf{k} \\ &= 150.57 \mathbf{i} + 86.93 \mathbf{j} - 46.59 \mathbf{k} \end{aligned}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \mathbf{i} + F_2 \cos \beta_2 \mathbf{j} + F_2 \cos \gamma_2 \mathbf{k}$$

$$\mathbf{F}_R = \{500 \mathbf{i}\} \text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$\mathbf{i}$  components :

$$500 = 150.57 + F_2 \cos \alpha_2$$

$$F_{1x} = F_2 \cos \alpha_2 = 349.43$$

$\mathbf{j}$  components :

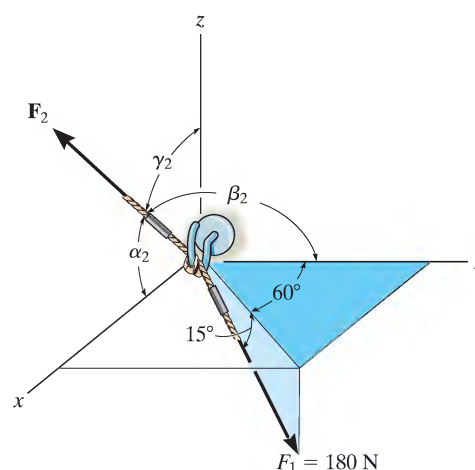
$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_{1y} = F_2 \cos \beta_2 = -86.93$$

$\mathbf{k}$  components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_{1z} = F_2 \cos \gamma_2 = 46.59$$



Thus,

$$F_2 = \sqrt{F_{1x}^2 + F_{1y}^2 + F_{1z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$$

$$F_2 = 363 \text{ N} \quad \text{Ans}$$

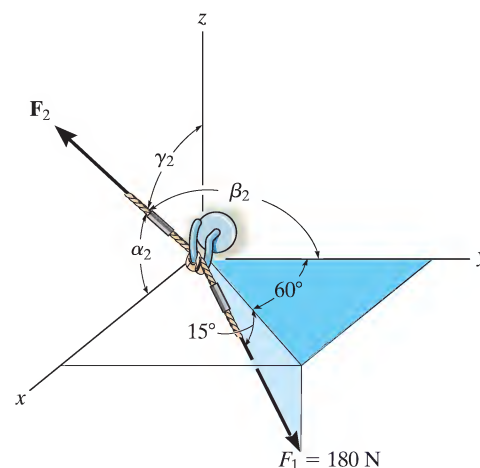
$$\alpha_2 = 15.8^\circ \quad \text{Ans}$$

$$\beta_2 = 104^\circ \quad \text{Ans}$$

$$\gamma_2 = 82.6^\circ \quad \text{Ans}$$

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•2-77. Determine the magnitude and coordinate direction angles of  $\mathbf{F}_2$  so that the resultant of the two forces is zero.



$$\begin{aligned}\mathbf{F}_1 &= (180 \cos 15^\circ) \sin 60^\circ \mathbf{i} + (180 \cos 15^\circ) \cos 60^\circ \mathbf{j} - 180 \sin 15^\circ \mathbf{k} \\ &= 150.57 \mathbf{i} + 86.93 \mathbf{j} - 46.59 \mathbf{k}\end{aligned}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \mathbf{i} + F_2 \cos \beta_2 \mathbf{j} + F_2 \cos \gamma_2 \mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

**i components :**

$$0 = 150.57 + F_2 \cos \alpha_2$$

$$F_2 \cos \alpha_2 = -150.57$$

**j components :**

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_2 \cos \beta_2 = -86.93$$

**k components :**

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_2 \cos \gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

**Solving,**

$$F_2 = 180 \text{ N} \quad \text{Ans}$$

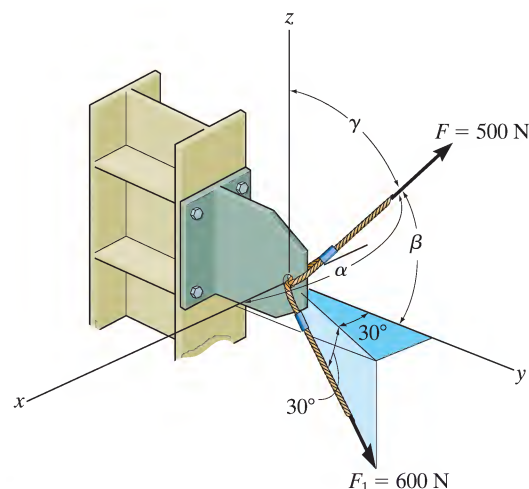
$$\alpha_2 = 147^\circ \quad \text{Ans}$$

$$\beta_2 = 119^\circ \quad \text{Ans}$$

$$\gamma_2 = 75.0^\circ \quad \text{Ans}$$

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**2-78.** If the resultant force acting on the bracket is directed along the positive  $y$  axis, determine the magnitude of the resultant force and the coordinate direction angles of  $\mathbf{F}$  so that  $\beta < 90^\circ$ .



**Force Vectors:** By resolving  $\mathbf{F}_1$  and  $\mathbf{F}$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs.  $a$  and  $b$ , respectively,  $\mathbf{F}_1$  and  $\mathbf{F}$  can be expressed in Cartesian vector form as

$$\begin{aligned}\mathbf{F}_1 &= 600 \cos 30^\circ \sin 30^\circ (+\mathbf{i}) + 600 \cos 30^\circ \cos 30^\circ (+\mathbf{j}) + 600 \sin 30^\circ (-\mathbf{k}) \\ &= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\} \text{ N} \\ \mathbf{F} &= 500 \cos \alpha \mathbf{i} + 500 \cos \beta \mathbf{j} + 500 \cos \gamma \mathbf{k}\end{aligned}$$

Since the resultant force  $\mathbf{F}_R$  is directed towards the positive  $y$  axis, then

$$\mathbf{F}_R = F_R \mathbf{j}$$

**Resultant Force:**

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F} \\ F_R \mathbf{j} &= (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500 \cos \alpha \mathbf{i} + 500 \cos \beta \mathbf{j} + 500 \cos \gamma \mathbf{k}) \\ F_R \mathbf{j} &= (259.81 + 500 \cos \alpha)\mathbf{i} + (450 + 500 \cos \beta)\mathbf{j} + (500 \cos \gamma - 300)\mathbf{k}\end{aligned}$$

Equating the  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  components,

$$0 = 259.81 + 500 \cos \alpha$$

$$\alpha = 121.31^\circ = 121^\circ$$

$$F_R = 450 + 500 \cos \beta$$

Ans.

$$0 = 500 \cos \gamma - 300$$

$$\gamma = 53.13^\circ = 53.1^\circ$$

Ans.

However, since  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ ,  $\alpha = 121.31^\circ$ , and  $\gamma = 53.13^\circ$ ,

$$\cos \beta = \pm \sqrt{1 - \cos^2 121.31^\circ - \cos^2 53.13^\circ} = \pm 0.6083$$

If we substitute  $\cos \beta = 0.6083$  into Eq. (1),

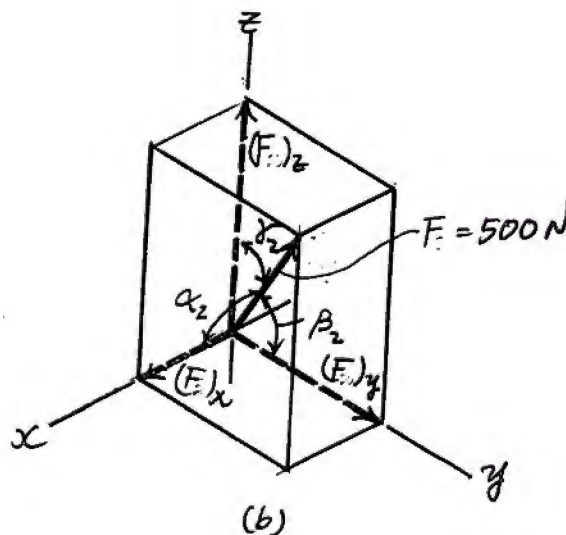
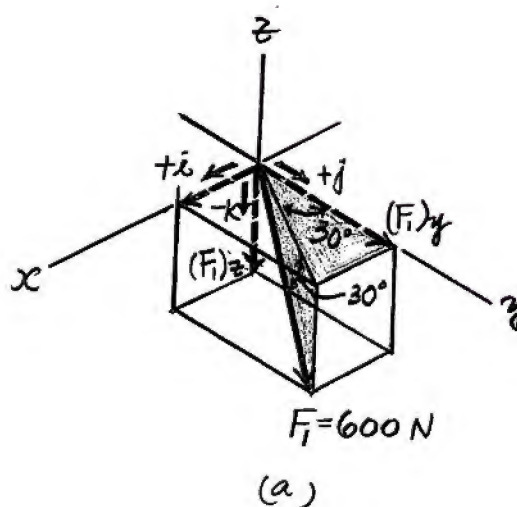
$$F_R = 450 + 500(0.6083) = 754 \text{ N}$$

Ans.

and

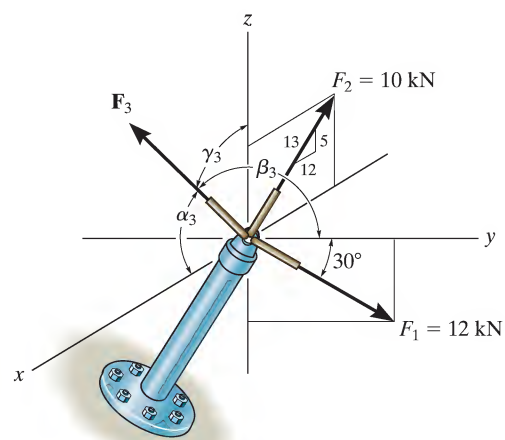
$$\beta = \cos^{-1}(0.6083) = 52.5^\circ$$

Ans.



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2-79. Specify the magnitude of  $\mathbf{F}_3$  and its coordinate direction angles  $\alpha_3, \beta_3, \gamma_3$  so that the resultant force  $\mathbf{F}_R = \{9\mathbf{j}\}$  kN.



$$\mathbf{F}_1 = 12 \cos 30^\circ \mathbf{j} - 12 \sin 30^\circ \mathbf{k} = 10.392 \mathbf{j} - 6 \mathbf{k}$$

$$\mathbf{F}_2 = -\frac{12}{13}(10) \mathbf{i} + \frac{5}{13}(10) \mathbf{k} = -9.231 \mathbf{i} + 3.846 \mathbf{k}$$

Require

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$9 \mathbf{j} = 10.392 \mathbf{j} - 6 \mathbf{k} - 9.231 \mathbf{i} + 3.846 \mathbf{k} + \mathbf{F}_3$$

$$\mathbf{F}_3 = 9.231 \mathbf{i} - 1.392 \mathbf{j} + 2.154 \mathbf{k}$$

Hence,

$$F_3 = \sqrt{(9.231)^2 + (-1.392)^2 + (2.154)^2}$$

$$F_3 = 9.581 \text{ kN} = 9.58 \text{ kN} \quad \text{Ans}$$

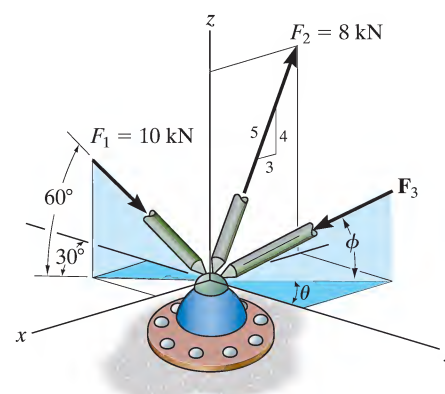
$$\alpha_3 = \cos^{-1}\left(\frac{9.231}{9.581}\right) = 15.5^\circ \quad \text{Ans}$$

$$\beta_3 = \cos^{-1}\left(\frac{-1.392}{9.581}\right) = 98.4^\circ \quad \text{Ans}$$

$$\gamma_3 = \cos^{-1}\left(\frac{2.154}{9.581}\right) = 77.0^\circ \quad \text{Ans}$$

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**\*2-80.** If  $F_3 = 9$  kN,  $\theta = 30^\circ$ , and  $\phi = 45^\circ$ , determine the magnitude and coordinate direction angles of the resultant force acting on the ball-and-socket joint.



**Force Vectors:** By resolving  $F_1$ ,  $F_2$  and  $F_3$  into their  $x$ ,  $y$ , and  $z$  components, as shown in Figs.

respectively,  $F_1$ ,  $F_2$  and  $F_3$  can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 10 \cos 60^\circ \sin 30^\circ (-\mathbf{i}) + 10 \cos 60^\circ \cos 30^\circ (+\mathbf{j}) + 10 \sin 60^\circ (-\mathbf{k})$$

$$= \{-2.5\mathbf{i} + 4.330\mathbf{j} - 8.660\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_2 = 8 \left( \frac{3}{5} \right) (-\mathbf{i}) + 0\mathbf{j} + 8 \left( \frac{4}{5} \right) (+\mathbf{k})$$

$$= \{-4.8\mathbf{i} + 6.4\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_3 = 9 \cos 45^\circ \sin 30^\circ (+\mathbf{i}) + 9 \cos 45^\circ \cos 30^\circ (-\mathbf{j}) + 9 \sin 45^\circ (-\mathbf{k})$$

$$= \{3.182\mathbf{i} - 5.511\mathbf{j} - 6.364\mathbf{k}\} \text{ kN}$$

**Resultant Force:** By adding  $F_1$ ,  $F_2$  and  $F_3$  vectorally, we obtain  $F_R$ . Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (-2.5\mathbf{i} + 4.330\mathbf{j} - 8.660\mathbf{k}) + (-4.8\mathbf{i} + 6.4\mathbf{k}) + (3.182\mathbf{i} - 5.511\mathbf{j} - 6.364\mathbf{k})$$

$$= \{-4.118\mathbf{i} - 1.181\mathbf{j} - 8.624\mathbf{k}\} \text{ kN}$$

The magnitude of  $F_R$  is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

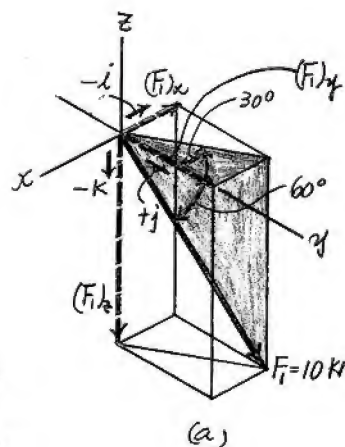
$$= \sqrt{(-4.118)^2 + (-1.181)^2 + (-8.624)^2} = 9.630 \text{ kN} = 9.63 \text{ kN} \quad \text{Ans.}$$

The coordinate direction angles of  $F_R$  are

$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{-4.118}{9.630} \right) = 115^\circ$$

$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{-1.181}{9.630} \right) = 97.0^\circ$$

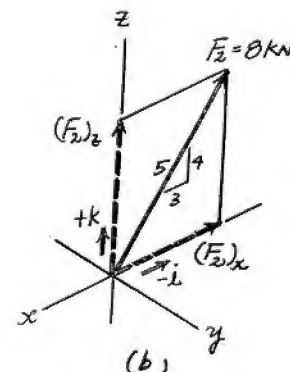
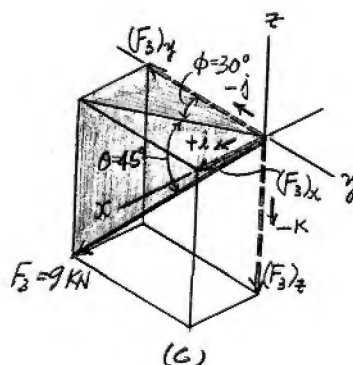
$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{-8.624}{9.630} \right) = 154^\circ$$



Ans.

Ans.

Ans.



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•2–81. The pole is subjected to the force  $\mathbf{F}$ , which has components acting along the  $x$ ,  $y$ ,  $z$  axes as shown. If the magnitude of  $\mathbf{F}$  is 3 kN,  $\beta = 30^\circ$ , and  $\gamma = 75^\circ$ , determine the magnitudes of its three components.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

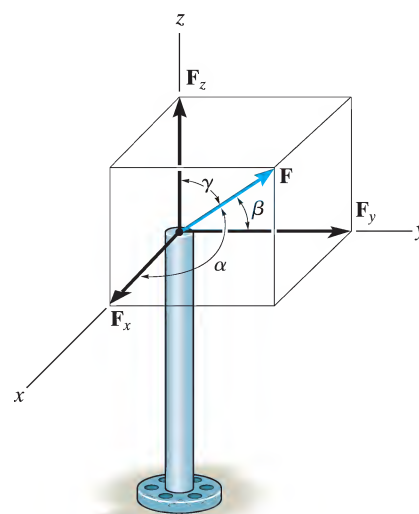
$$\cos^2 \alpha + \cos^2 30^\circ + \cos^2 75^\circ = 1$$

$$\alpha = 64.67^\circ$$

$$F_x = 3 \cos 64.67^\circ = 1.28 \text{ kN} \quad \text{Ans}$$

$$F_y = 3 \cos 30^\circ = 2.60 \text{ kN} \quad \text{Ans}$$

$$F_z = 3 \cos 75^\circ = 0.776 \text{ kN} \quad \text{Ans}$$



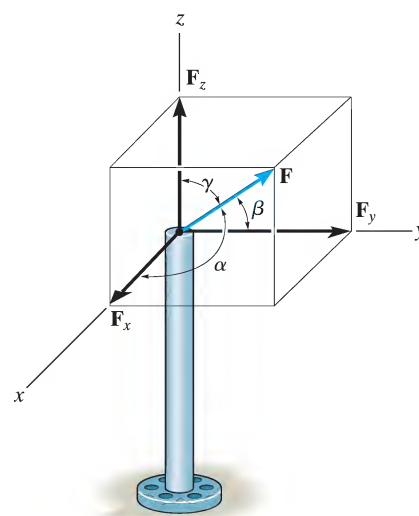
2–82. The pole is subjected to the force  $\mathbf{F}$  which has components  $F_x = 1.5 \text{ kN}$  and  $F_z = 1.25 \text{ kN}$ . If  $\beta = 75^\circ$ , determine the magnitudes of  $\mathbf{F}$  and  $F_y$ .

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75^\circ + \left(\frac{1.25}{F}\right)^2 = 1$$

$$F = 2.02 \text{ kN} \quad \text{Ans}$$

$$F_y = 2.02 \cos 75^\circ = 0.523 \text{ kN} \quad \text{Ans}$$



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2-83. Three forces act on the ring. If the resultant force  $\mathbf{F}_R$  has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force  $\mathbf{F}_3$ .

**Cartesian Vector Notation :**

$$\mathbf{F}_R = 120 \{ \cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k} \} \text{ N}$$

$$= \{ 42.43 \mathbf{i} + 73.48 \mathbf{j} + 84.85 \mathbf{k} \} \text{ N}$$

$$\mathbf{F}_1 = 80 \left\{ \frac{4}{5} \mathbf{i} + \frac{3}{5} \mathbf{k} \right\} \text{ N} = \{ 64.0 \mathbf{i} + 48.0 \mathbf{k} \} \text{ N}$$

$$\mathbf{F}_2 = \{ -110 \mathbf{k} \} \text{ N}$$

$$\mathbf{F}_3 = \{ F_{3x} \mathbf{i} + F_{3y} \mathbf{j} + F_{3z} \mathbf{k} \} \text{ N}$$

**Resultant Force :**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{ 42.43 \mathbf{i} + 73.48 \mathbf{j} + 84.85 \mathbf{k} \}$$

$$= \{ (64.0 + F_{3x}) \mathbf{i} + F_{3y} \mathbf{j} + (48.0 - 110 + F_{3z}) \mathbf{k} \}$$

Equating i, j and k components, we have

$$64.0 + F_{3x} = 42.43 \quad F_{3x} = -21.57 \text{ N}$$

$$F_{3y} = 73.48 \text{ N}$$

$$48.0 - 110 + F_{3z} = 84.85 \quad F_{3z} = 146.85 \text{ N}$$

The magnitude of force  $\mathbf{F}_3$  is

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2 + F_{3z}^2}$$

$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$

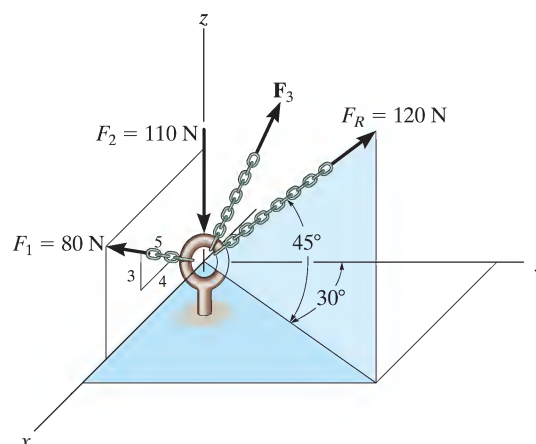
$$= 165.62 \text{ N} = 166 \text{ N} \quad \text{Ans}$$

The coordinate direction angles for  $\mathbf{F}_3$  are

$$\cos \alpha = \frac{F_{3x}}{F_3} = \frac{-21.57}{165.62} \quad \alpha = 97.5^\circ \quad \text{Ans}$$

$$\cos \beta = \frac{F_{3y}}{F_3} = \frac{73.48}{165.62} \quad \beta = 63.7^\circ \quad \text{Ans}$$

$$\cos \gamma = \frac{F_{3z}}{F_3} = \frac{146.85}{165.62} \quad \gamma = 27.5^\circ \quad \text{Ans}$$



\*2-84. Determine the coordinate direction angles of  $\mathbf{F}_1$  and  $\mathbf{F}_R$ .

**Unit Vector of  $\mathbf{F}_1$  and  $\mathbf{F}_R$  :**

$$\mathbf{u}_{F_1} = \frac{4}{5} \mathbf{i} + \frac{3}{5} \mathbf{k} = 0.8 \mathbf{i} + 0.6 \mathbf{k}$$

$$\mathbf{u}_R = \cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}$$

$$= 0.3536 \mathbf{i} + 0.6124 \mathbf{j} + 0.7071 \mathbf{k}$$

Thus, the coordinate direction angles  $\mathbf{F}_1$  and  $\mathbf{F}_R$  are

$$\cos \alpha_{F_1} = 0.8 \quad \alpha_{F_1} = 36.9^\circ \quad \text{Ans}$$

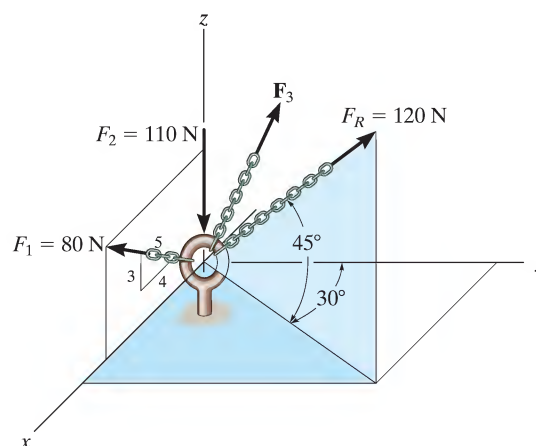
$$\cos \beta_{F_1} = 0 \quad \beta_{F_1} = 90.0^\circ \quad \text{Ans}$$

$$\cos \gamma_{F_1} = 0.6 \quad \gamma_{F_1} = 53.1^\circ \quad \text{Ans}$$

$$\cos \alpha_R = 0.3536 \quad \alpha_R = 69.3^\circ \quad \text{Ans}$$

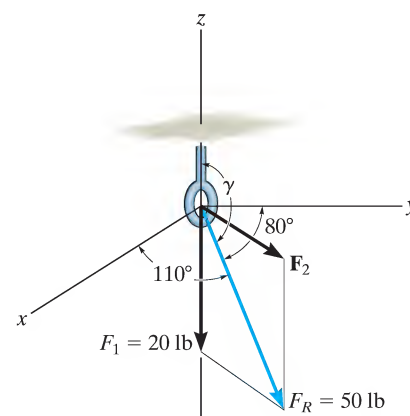
$$\cos \beta_R = 0.6124 \quad \beta_R = 52.2^\circ \quad \text{Ans}$$

$$\cos \gamma_R = 0.7071 \quad \gamma_R = 45.0^\circ \quad \text{Ans}$$



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•2–85. Two forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  act on the bolt. If the resultant force  $\mathbf{F}_R$  has a magnitude of 50 lb and coordinate direction angles  $\alpha = 110^\circ$  and  $\beta = 80^\circ$ , as shown, determine the magnitude of  $\mathbf{F}_2$  and its coordinate direction angles.



$$(1)^2 = \cos^2 110^\circ + \cos^2 80^\circ + \cos^2 \gamma$$

$$\gamma = 157.44^\circ$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$50 \cos 110^\circ = (\mathbf{F}_2)_x$$

$$50 \cos 80^\circ = (\mathbf{F}_2)_y$$

$$50 \cos 157.44^\circ = (\mathbf{F}_2)_z - 20$$

$$(\mathbf{F}_2)_x = -17.10$$

$$(\mathbf{F}_2)_y = 8.68$$

$$(\mathbf{F}_2)_z = -26.17$$

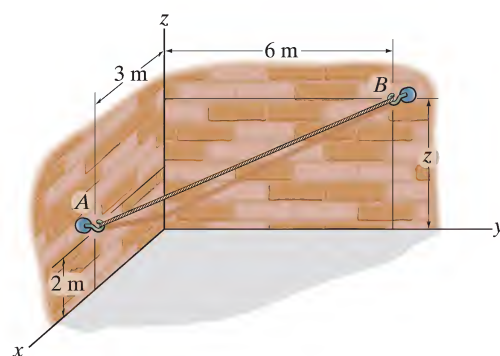
$$F_2 = \sqrt{(-17.10)^2 + (8.68)^2 + (-26.17)^2} = 32.4 \text{ lb} \quad \text{Ans}$$

$$\alpha_2 = \cos^{-1}\left(\frac{-17.10}{32.4}\right) = 122^\circ \quad \text{Ans}$$

$$\beta_2 = \cos^{-1}\left(\frac{8.68}{32.4}\right) = 74.5^\circ \quad \text{Ans}$$

$$\gamma_2 = \cos^{-1}\left(\frac{-26.17}{32.4}\right) = 144^\circ \quad \text{Ans}$$

2–86. Determine the position vector  $\mathbf{r}$  directed from point  $A$  to point  $B$  and the length of cord  $AB$ . Take  $z = 4 \text{ m}$ .



**Position Vector:** The coordinates for points  $A$  and  $B$  are  $A(3, 0, 2) \text{ m}$  and  $B(0, 6, 4) \text{ m}$ , respectively. Thus,

$$\begin{aligned} \mathbf{r}_{AB} &= (0 - 3)\mathbf{i} + (6 - 0)\mathbf{j} + (4 - 2)\mathbf{k} \\ &= \{-3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m} \quad \text{Ans.} \end{aligned}$$

The length of cord  $AB$  is

$$r_{AB} = \sqrt{(-3)^2 + 6^2 + 2^2} = 7 \text{ m} \quad \text{Ans.}$$

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2-87. If the cord  $AB$  is 7.5 m long, determine the coordinate position  $+z$  of point  $B$

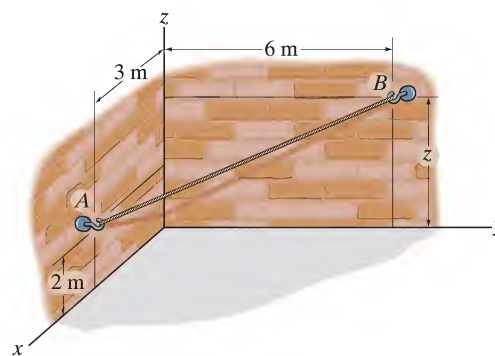
**Position Vector:** The coordinates for points  $A$  and  $B$  are  $A(3, 0, 2)$  m and  $B(0, 6, z)$  m, respectively. Thus,

$$\begin{aligned}\mathbf{r}_{AB} &= (0 - 3)\mathbf{i} + (6 - 0)\mathbf{j} + (z - 2)\mathbf{k} \\ &= \{-3\mathbf{i} + 6\mathbf{j} + (z - 2)\mathbf{k}\} \text{ m}\end{aligned}$$

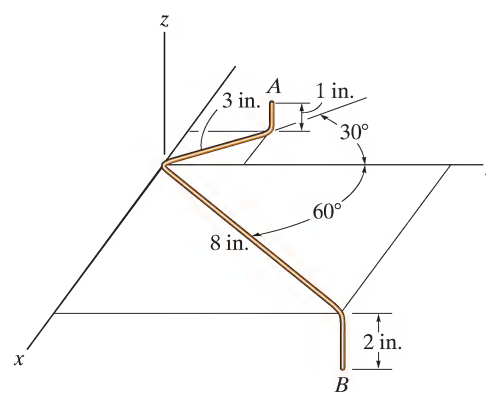
Since the length of cord is equal to the magnitude of  $\mathbf{r}_{AB}$ , then

$$\begin{aligned}r_{AB} &= 7.5 = \sqrt{(-3)^2 + 6^2 + (z - 2)^2} \\ 56.25 &= 45 + (z - 2)^2 \\ z - 2 &= \pm 3.354 \\ z &= 5.35 \text{ m}\end{aligned}$$

**Ans.**



\*2-88. Determine the distance between the end points  $A$  and  $B$  on the wire by first formulating a position vector from  $A$  to  $B$  and then determining its magnitude.



$$\mathbf{r}_{AB} = (8 \sin 60^\circ - 3 \sin 30^\circ)\mathbf{i} + (8 \cos 60^\circ - 3 \cos 30^\circ)\mathbf{j} + (-2 - 1)\mathbf{k}$$

$$\mathbf{r}_{AB} = (8.428\mathbf{i} + 1.402\mathbf{j} - 3\mathbf{k}) \text{ in.}$$

$$r_{AB} = \sqrt{(8.428)^2 + (1.402)^2 + (-3)^2} = 9.06 \text{ in.} \quad \text{Ans}$$

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•2–89. Determine the magnitude and coordinate direction angles of the resultant force acting at A.

**Unit Vectors:** The coordinate points A, B, and C are shown in Fig. a. Thus,

$$\begin{aligned}\mathbf{u}_B &= \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(3-0)\mathbf{i} + (-3-0)\mathbf{j} + (2.5-4)\mathbf{k}}{\sqrt{(3-0)^2 + (-3-0)^2 + (2.5-4)^2}} \\ &= \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k} \\ \mathbf{u}_C &= \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2-0)\mathbf{i} + (4-0)\mathbf{j} + (0-4)\mathbf{k}}{\sqrt{(2-0)^2 + (4-0)^2 + (0-4)^2}} \\ &= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\end{aligned}$$

**Force Vectors:** Multiplying the magnitude of the force with its unit vector, we have

$$\mathbf{F}_B = F_B \mathbf{u}_B = 600 \left( \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k} \right) = \{400\mathbf{i} - 400\mathbf{j} - 200\mathbf{k}\} \text{ lb} \quad \text{Ans.}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 750 \left( \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right) = \{250\mathbf{i} + 500\mathbf{j} - 500\mathbf{k}\} \text{ lb} \quad \text{Ans.}$$

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = 400\mathbf{i} - 400\mathbf{j} - 200\mathbf{k} + 250\mathbf{i} + 500\mathbf{j} - 500\mathbf{k}$$

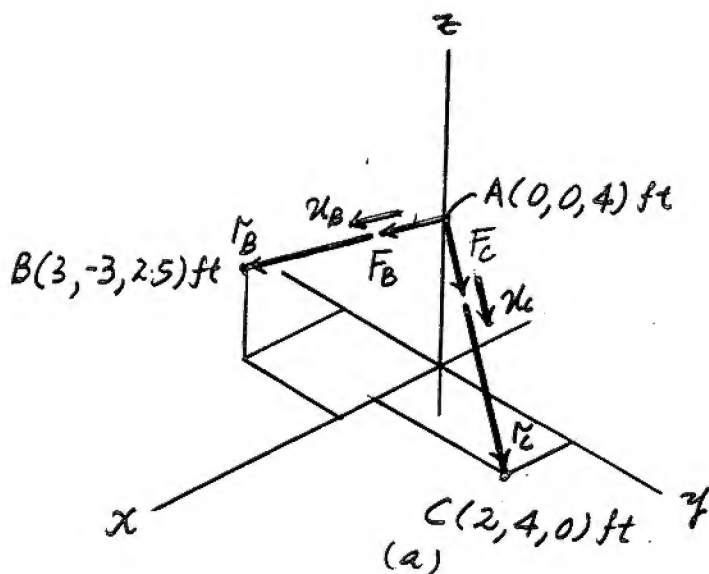
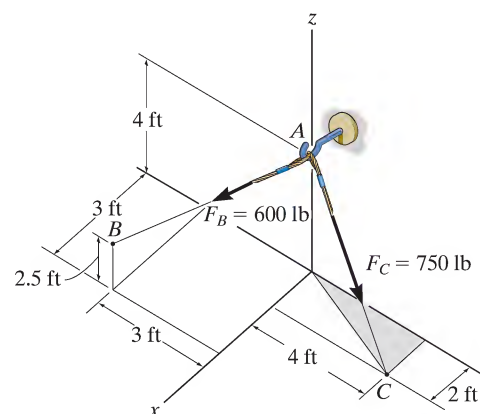
$$\mathbf{F}_R = \{650\mathbf{i} + 100\mathbf{j} - 700\mathbf{k}\} \text{ lb}$$

$$F_R = \sqrt{650^2 + 100^2 + (-700)^2} = 960 \text{ lb} \quad \text{Ans.}$$

$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{650}{960} \right) = 47.4^\circ \quad \text{Ans.}$$

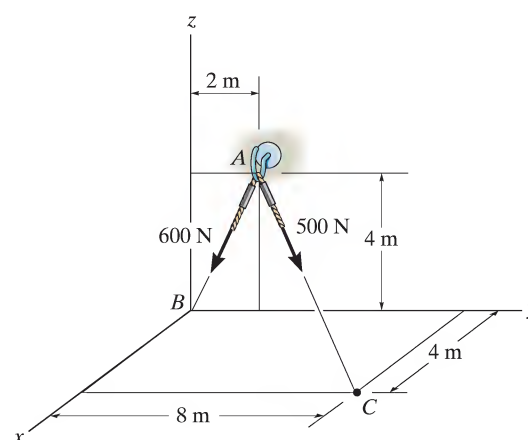
$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{100}{960} \right) = 84.0^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{-700}{960} \right) = 137^\circ \quad \text{Ans.}$$



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**2-90.** Determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{r}_{AB} = \{-2\mathbf{j} - 4\mathbf{k}\}\text{m}; \quad r_{AB} = 4.472\text{ m}$$

$$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = -0.447\mathbf{j} - 0.894\mathbf{k}$$

$$\mathbf{F}_{AB} = 600\mathbf{u}_{AB} = \{-268.33\mathbf{j} - 536.66\mathbf{k}\}\text{N}$$

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}\}\text{m}; \quad r_{AC} = 8.246\text{ m}$$

$$\mathbf{u}_{AC} = \left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 0.485\mathbf{i} + 0.728\mathbf{j} - 0.485\mathbf{k}$$

$$\mathbf{F}_{AC} = 500\mathbf{u}_{AC} = \{242.54\mathbf{i} + 363.80\mathbf{j} - 242.54\mathbf{k}\}\text{N}$$

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$\mathbf{F}_R = \{242.54\mathbf{i} + 95.47\mathbf{j} - 779.20\mathbf{k}\}$$

$$F_R = \sqrt{(242.54)^2 + (95.47)^2 + (-779.20)^2} = 821.64 = 822\text{ N} \quad \text{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{242.54}{821.64}\right) = 72.8^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{95.47}{821.64}\right) = 83.3^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-779.20}{821.64}\right) = 162^\circ \quad \text{Ans}$$

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2-91. Determine the magnitude and coordinate direction angles of the resultant force acting at A.

**Force Vectors:** The unit vectors  $\mathbf{u}_B$  and  $\mathbf{u}_C$  of  $\mathbf{F}_B$  and  $\mathbf{F}_C$  must be determined first. From Fig. a

$$\begin{aligned}\mathbf{u}_B &= \frac{\mathbf{r}_B}{r_B} = \frac{(4.5 \sin 45^\circ - 0)\mathbf{i} + (-4.5 \cos 45^\circ - 0)\mathbf{j} + (0 - 6)\mathbf{k}}{\sqrt{(4.5 \sin 45^\circ - 0)^2 + (-4.5 \cos 45^\circ - 0)^2 + (0 - 6)^2}} \\ &= 0.4243\mathbf{i} - 0.4243\mathbf{j} - 0.8\mathbf{k} \\ \mathbf{u}_C &= \frac{\mathbf{r}_C}{r_C} = \frac{(-3 - 0)\mathbf{i} + (-6 - 0)\mathbf{j} + (0 - 6)\mathbf{k}}{\sqrt{(-3 - 0)^2 + (-6 - 0)^2 + (0 - 6)^2}} \\ &= -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\end{aligned}$$

Thus, the force vectors  $\mathbf{F}_B$  and  $\mathbf{F}_C$  are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 900(0.4243\mathbf{i} - 0.4243\mathbf{j} - 0.8\mathbf{k}) = \{381.84\mathbf{i} - 381.84\mathbf{j} - 720\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} - 400\mathbf{j} - 400\mathbf{k}\} \text{ N}$$

**Resultant Force:**

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C = (381.84\mathbf{i} - 381.84\mathbf{j} - 720\mathbf{k}) + (-200\mathbf{i} - 400\mathbf{j} - 400\mathbf{k}) \\ &= \{181.84\mathbf{i} - 781.84\mathbf{j} - 1120\mathbf{k}\} \text{ N}\end{aligned}$$

The magnitude of  $\mathbf{F}_R$  is

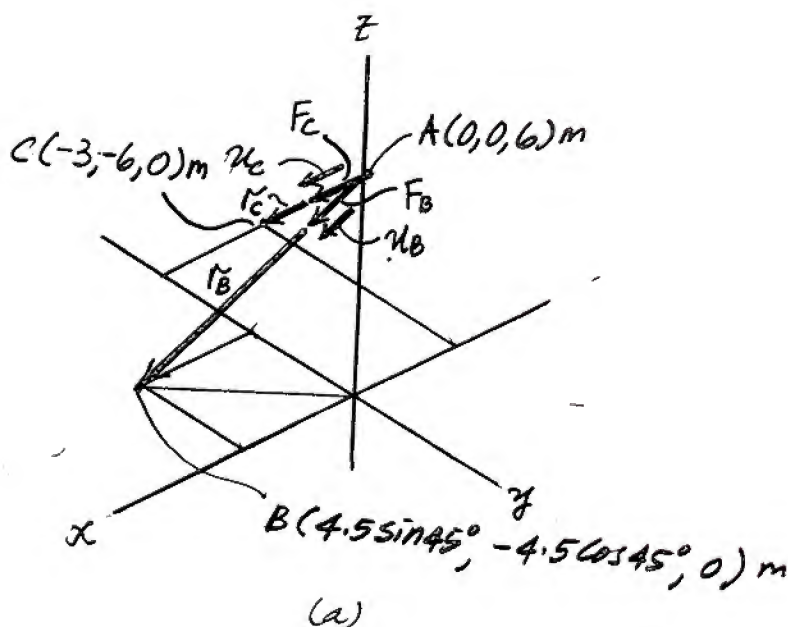
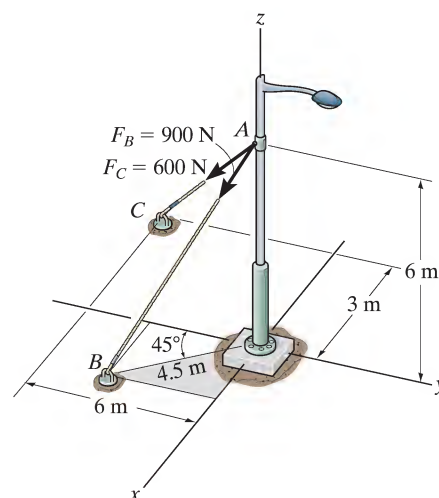
$$\begin{aligned}F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(181.84)^2 + (-781.84)^2 + (-1120)^2} = 1377.95 \text{ N} = 1.38 \text{ kN} \quad \text{Ans.}\end{aligned}$$

The coordinate direction angles of  $\mathbf{F}_R$  are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{181.84}{1377.95}\right) = 82.4^\circ \quad \text{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-781.84}{1377.95}\right) = 125^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1120}{1377.95}\right) = 144^\circ \quad \text{Ans.}$$



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\*2-92. Determine the magnitude and coordinate direction angles of the resultant force.

$$\mathbf{F}_1 = -100\left(\frac{3}{5}\right) \sin 40^\circ \mathbf{i} + 100\left(\frac{3}{5}\right) \cos 40^\circ \mathbf{j} - 100\left(\frac{4}{5}\right) \mathbf{k}$$

$$= \{-38.567 \mathbf{i} + 45.963 \mathbf{j} - 80 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = 81 \text{ lb} \left( \frac{4}{9} \mathbf{i} - \frac{7}{9} \mathbf{j} - \frac{4}{9} \mathbf{k} \right)$$

$$= \{36 \mathbf{i} - 63 \mathbf{j} - 36 \mathbf{k}\} \text{ lb}$$

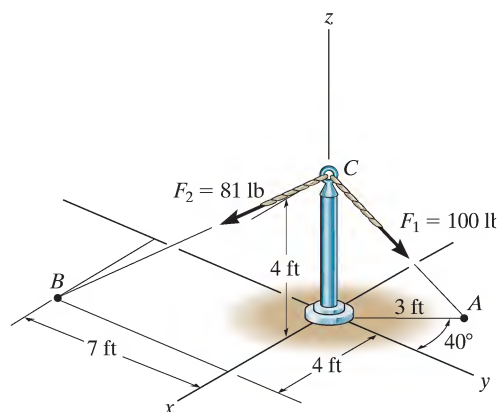
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-2.567 \mathbf{i} - 17.04 \mathbf{j} - 116.0 \mathbf{k}\} \text{ lb}$$

$$F_R = \sqrt{(-2.567)^2 + (-17.04)^2 + (-116.0)^2} = 117.27 \text{ lb} = 117 \text{ lb} \quad \text{Ans}$$

$$\alpha = \cos^{-1} \left( \frac{-2.567}{117.27} \right) = 91.3^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1} \left( \frac{-17.04}{117.27} \right) = 98.4^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1} \left( \frac{-116.0}{117.27} \right) = 172^\circ \quad \text{Ans}$$



•2-93. The chandelier is supported by three chains which are concurrent at point  $O$ . If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.

$$\mathbf{F}_A = 60 \frac{(4 \cos 30^\circ \mathbf{i} - 4 \sin 30^\circ \mathbf{j} - 6 \mathbf{k})}{\sqrt{(4 \cos 30^\circ)^2 + (-4 \sin 30^\circ)^2 + (-6)^2}}$$

$$= \{28.8 \mathbf{i} - 16.6 \mathbf{j} - 49.9 \mathbf{k}\} \text{ lb} \quad \text{Ans}$$

$$\mathbf{F}_B = 60 \frac{(-4 \cos 30^\circ \mathbf{i} - 4 \sin 30^\circ \mathbf{j} - 6 \mathbf{k})}{\sqrt{(-4 \cos 30^\circ)^2 + (-4 \sin 30^\circ)^2 + (-6)^2}}$$

$$= \{-28.8 \mathbf{i} - 16.6 \mathbf{j} - 49.9 \mathbf{k}\} \text{ lb} \quad \text{Ans}$$

$$\mathbf{F}_C = 60 \frac{(4 \mathbf{j} - 6 \mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3 \mathbf{j} - 49.9 \mathbf{k}\} \text{ lb} \quad \text{Ans}$$

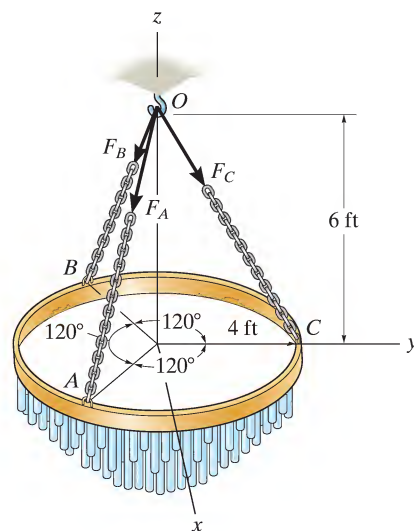
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8 \mathbf{k}\} \text{ lb}$$

$$F_R = 150 \text{ lb} \quad \text{Ans}$$

$$\alpha = 90^\circ \quad \text{Ans}$$

$$\beta = 90^\circ \quad \text{Ans}$$

$$\gamma = 180^\circ \quad \text{Ans}$$



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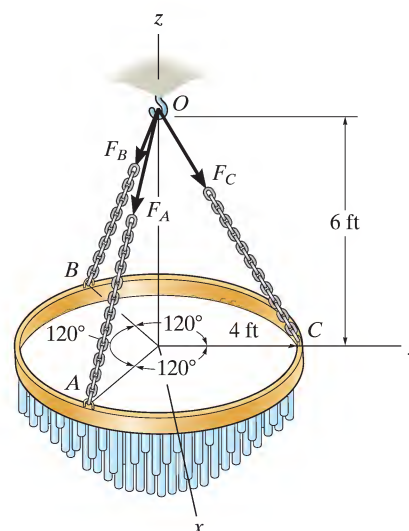
2-94. The chandelier is supported by three chains which are concurrent at point  $O$ . If the resultant force at  $O$  has a magnitude of 130 lb and is directed along the negative  $z$  axis, determine the force in each chain.

$$\mathbf{F}_C = F \frac{(4\mathbf{j} - 6\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547 F\mathbf{j} - 0.8321 F\mathbf{k}$$

$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

$$F_{Rz} = \Sigma F_z; \quad 130 = 3(0.8321 F)$$

$$F = 52.1 \text{ lb} \quad \text{Ans}$$



2-95. Express force  $\mathbf{F}$  as a Cartesian vector; then determine its coordinate direction angles.

**Unit Vector :** The coordinates of point  $A$  are

$$A(-10\cos 70^\circ \sin 30^\circ, 10\cos 70^\circ \cos 30^\circ, 10\sin 70^\circ) \text{ ft} \\ = A(-1.710, 2.962, 9.397) \text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(5 - (-1.710))\mathbf{i} + (-7 - 2.962)\mathbf{j} + (0 - 9.397)\mathbf{k}\} \text{ ft} \\ = \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\} \text{ ft}$$

$$r_{AB} = \sqrt{6.710^2 + (-9.962)^2 + (-9.397)^2} = 15.250 \text{ ft}$$

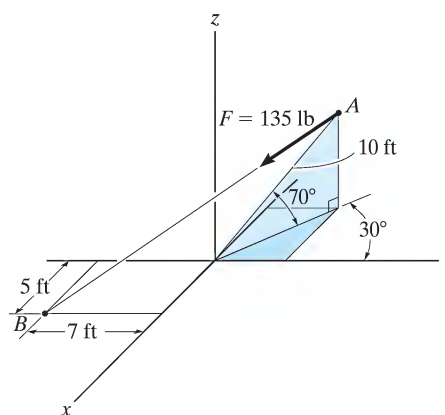
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}}{15.250} \\ = 0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}$$

**Force Vector :**

$$\mathbf{F} = F\mathbf{u}_{AB} = 135\{0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}\} \text{ lb} \\ = \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\} \text{ lb} \quad \text{Ans}$$

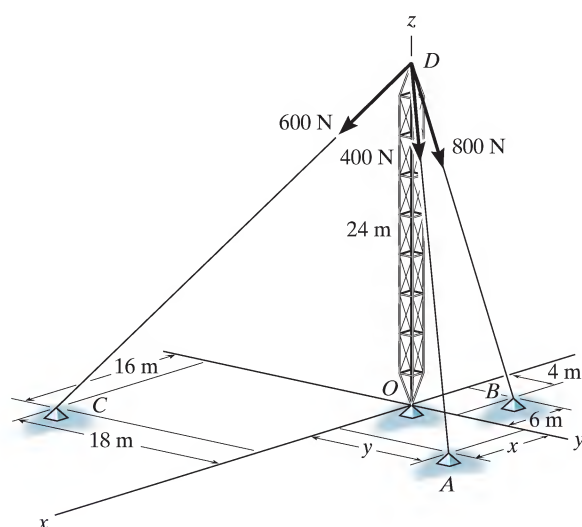
**Coordinate Direction Angles :** From the unit vector  $\mathbf{u}_{AB}$  obtained above, we have

$$\begin{array}{lll} \cos \alpha = 0.4400 & \alpha = 63.9^\circ & \text{Ans} \\ \cos \beta = -0.6532 & \beta = 131^\circ & \text{Ans} \\ \cos \gamma = -0.6162 & \gamma = 128^\circ & \text{Ans} \end{array}$$



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**\*2-96.** The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles  $\alpha$ ,  $\beta$ ,  $\gamma$  of the resultant force. Take  $x = 20$  m,  $y = 15$  m.



$$\mathbf{F}_{DA} = 400 \left( \frac{20}{34.66} \mathbf{i} + \frac{15}{34.66} \mathbf{j} - \frac{24}{34.66} \mathbf{k} \right) \text{ N}$$

$$\mathbf{F}_{DB} = 800 \left( \frac{-6}{25.06} \mathbf{i} + \frac{4}{25.06} \mathbf{j} - \frac{24}{25.06} \mathbf{k} \right) \text{ N}$$

$$\mathbf{F}_{DC} = 600 \left( \frac{16}{34} \mathbf{i} - \frac{18}{34} \mathbf{j} - \frac{24}{34} \mathbf{k} \right) \text{ N}$$

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} \\ &= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\} \text{ N} \end{aligned}$$

$$\begin{aligned} F_R &= \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2} \\ &= 1501.66 \text{ N} = 1.50 \text{ kN} \quad \text{Ans} \end{aligned}$$

$$\alpha = \cos^{-1} \left( \frac{321.66}{1501.66} \right) = 77.6^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1} \left( \frac{-16.82}{1501.66} \right) = 90.6^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1} \left( \frac{-1466.71}{1501.66} \right) = 168^\circ \quad \text{Ans}$$

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•2–97. The door is held opened by means of two chains. If the tension in  $AB$  and  $CD$  is  $F_A = 300$  N and  $F_C = 250$  N, respectively, express each of these forces in Cartesian vector form.

**Unit Vector :** First determine the position vector  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{CD}$ . The coordinates of points  $A$  and  $C$  are

$$A[0, -(1 + 1.5\cos 30^\circ), 1.5\sin 30^\circ] \text{ m} = A(0, -2.299, 0.750) \text{ m}$$

$$C[-2.50, -(1 + 1.5\cos 30^\circ), 1.5\sin 30^\circ] \text{ m} = C(-2.50, -2.299, 0.750) \text{ m}$$

Then

$$\mathbf{r}_{AB} = \{(0-0)\mathbf{i} + [0-(-2.299)]\mathbf{j} + (0-0.750)\mathbf{k}\} \text{ m}$$

$$= \{2.299\mathbf{j} - 0.750\mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{2.299^2 + (-0.750)^2} = 2.418 \text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.299\mathbf{j} - 0.750\mathbf{k}}{2.418} = 0.9507\mathbf{j} - 0.3101\mathbf{k}$$

$$\mathbf{r}_{CD} = \{[-0.5-(-2.5)]\mathbf{i} + [0-(-2.299)]\mathbf{j} + (0-0.750)\mathbf{k}\} \text{ m}$$

$$= \{2.00\mathbf{i} + 2.299\mathbf{j} - 0.750\mathbf{k}\} \text{ m}$$

$$r_{CD} = \sqrt{2.00^2 + 2.299^2 + (-0.750)^2} = 3.138 \text{ m}$$

$$\mathbf{u}_{CD} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{2.00\mathbf{i} + 2.299\mathbf{j} - 0.750\mathbf{k}}{3.138} = 0.6373\mathbf{i} + 0.7326\mathbf{j} - 0.2390\mathbf{k}$$

**Force Vector :**

$$\mathbf{F}_A = F_A \mathbf{u}_{AB} = 300(0.9507\mathbf{j} - 0.3101\mathbf{k}) \text{ N}$$

$$= \{285.21\mathbf{j} - 93.04\mathbf{k}\} \text{ N}$$

$$= \{285\mathbf{j} - 93.0\mathbf{k}\} \text{ N}$$

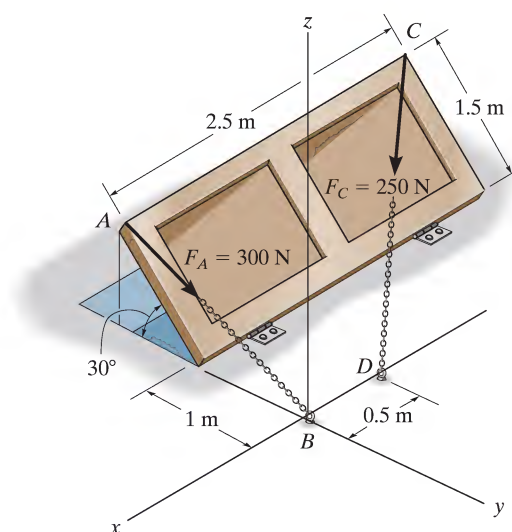
Ans

$$\mathbf{F}_C = F_C \mathbf{u}_{CD} = 250(0.6373\mathbf{i} + 0.7326\mathbf{j} - 0.2390\mathbf{k}) \text{ N}$$

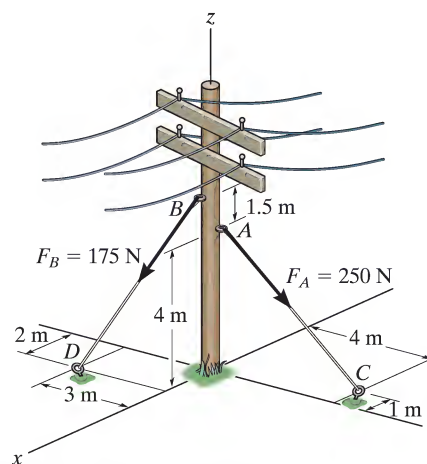
$$= \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\} \text{ N}$$

$$= \{159\mathbf{i} + 183\mathbf{j} - 59.7\mathbf{k}\} \text{ N}$$

Ans



2–98. The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.



**Unit Vector :**

$$\mathbf{r}_{AC} = \{(-1-0)\mathbf{i} + (4-0)\mathbf{j} + (0-4)\mathbf{k}\} \text{ m} = \{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(-1)^2 + 4^2 + (-4)^2} = 5.745 \text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{5.745} = -0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}$$

$$\mathbf{r}_{BD} = \{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-5.5)\mathbf{k}\} \text{ m} = \{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}\} \text{ m}$$

$$r_{BD} = \sqrt{2^2 + (-3)^2 + (-5.5)^2} = 6.576 \text{ m}$$

$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}$$

**Force Vector :**

$$\mathbf{F}_A = F_A \mathbf{u}_{AC} = 250(-0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}) \text{ N}$$

$$= \{-43.52\mathbf{i} + 174.08\mathbf{j} - 174.08\mathbf{k}\} \text{ N}$$

$$= \{-43.5\mathbf{i} + 174\mathbf{j} - 174\mathbf{k}\} \text{ N}$$

Ans

$$\mathbf{F}_B = F_B \mathbf{u}_{BD} = 175(0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}) \text{ N}$$

$$= \{53.22\mathbf{i} - 79.83\mathbf{j} - 146.36\mathbf{k}\} \text{ N}$$

$$= \{53.2\mathbf{i} - 79.8\mathbf{j} - 146\mathbf{k}\} \text{ N}$$

Ans

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**2-99.** Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point  $A$  towards  $O$ , determine the magnitudes of the resultant force and forces  $F_B$  and  $F_C$ . Set  $x = 3$  m and  $z = 2$  m.

**Force Vectors:** The unit vectors  $\mathbf{u}_B$  and  $\mathbf{u}_C$  must be determined first. From Fig.  $a$ ,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i} + (0-6)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-6)^2 + (3-0)^2}} = -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (0-6)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(3-0)^2 + (0-6)^2 + (2-0)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

Thus, the force vectors  $\mathbf{F}_B$  and  $\mathbf{F}_C$  are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = -\frac{2}{7}F_B \mathbf{i} - \frac{6}{7}F_B \mathbf{j} + \frac{3}{7}F_B \mathbf{k}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = \frac{3}{7}F_C \mathbf{i} - \frac{6}{7}F_C \mathbf{j} + \frac{2}{7}F_C \mathbf{k}$$

Since the resultant force  $\mathbf{F}_R$  is directed along the negative  $y$  axis, and the load  $\mathbf{W}$  is directed along the  $z$  axis, these two forces can be written as

$$\mathbf{F}_R = -F_R \mathbf{j} \quad \text{and} \quad \mathbf{W} = [-1500\mathbf{k}] \text{ N}$$

**Resultant Force:** The vector addition of  $\mathbf{F}_B$ ,  $\mathbf{F}_C$ , and  $\mathbf{W}$  is equal to  $\mathbf{F}_R$ . Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R \mathbf{j} = \left(-\frac{2}{7}F_B \mathbf{i} - \frac{6}{7}F_B \mathbf{j} + \frac{3}{7}F_B \mathbf{k}\right) + \left(\frac{3}{7}F_C \mathbf{i} - \frac{6}{7}F_C \mathbf{j} + \frac{2}{7}F_C \mathbf{k}\right) + (-1500\mathbf{k})$$

$$-F_R \mathbf{j} = \left(-\frac{2}{7}F_B + \frac{3}{7}F_C\right)\mathbf{i} + \left(-\frac{6}{7}F_B - \frac{6}{7}F_C\right)\mathbf{j} + \left(\frac{3}{7}F_B + \frac{2}{7}F_C - 1500\right)\mathbf{k}$$

Equating the  $i$ ,  $j$ , and  $k$  components,

$$0 = -\frac{2}{7}F_B + \frac{3}{7}F_C \quad (1)$$

$$-F_R = -\frac{6}{7}F_B - \frac{6}{7}F_C \quad (2)$$

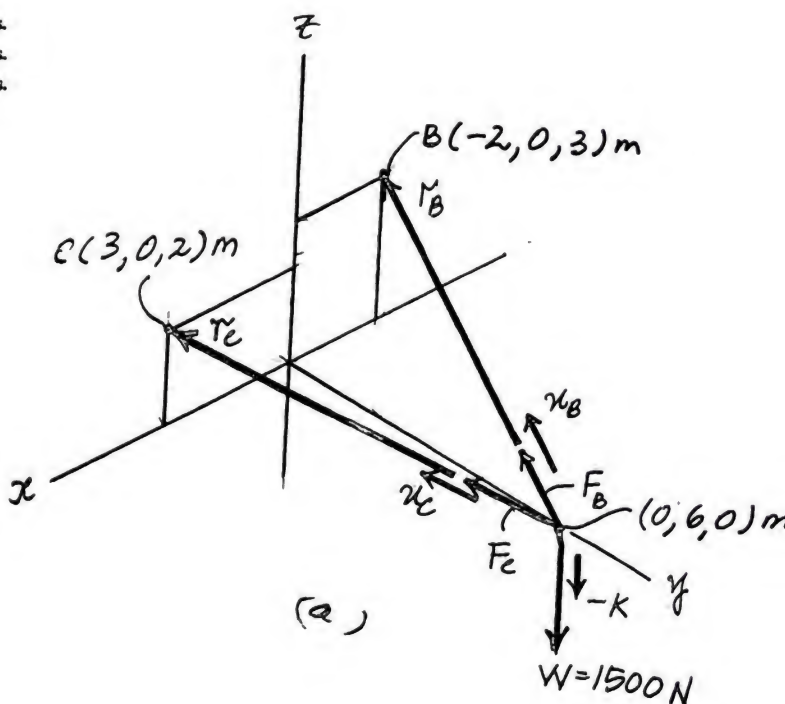
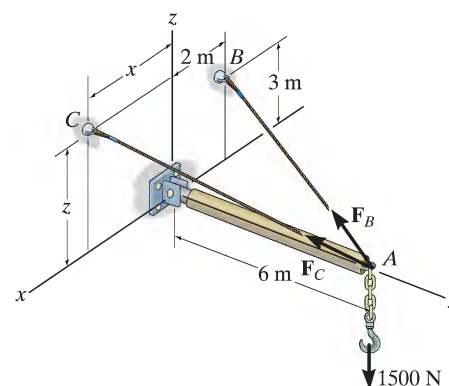
$$0 = \frac{3}{7}F_B + \frac{2}{7}F_C - 1500 \quad (3)$$

Solving Eqs. (1), (2), and (3) yields

$$F_C = 1615.38 \text{ N} = 1.62 \text{ kN} \quad \text{Ans.}$$

$$F_B = 2423.08 \text{ N} = 2.42 \text{ kN} \quad \text{Ans.}$$

$$F_R = 3461.53 \text{ N} = 3.46 \text{ kN} \quad \text{Ans.}$$



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**\*2-100.** Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point  $A$  towards  $O$ , determine the values of  $x$  and  $z$  for the coordinates of point  $C$  and the magnitude of the resultant force. Set  $F_B = 1610$  N and  $F_C = 2400$  N.

**Force Vectors:** From Fig.  $a$ ,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i} + (0-6)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-6)^2 + (3-0)^2}} = -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i} + (0-6)\mathbf{j} + (z-0)\mathbf{k}}{\sqrt{(x-0)^2 + (0-6)^2 + (z-0)^2}} = \frac{x}{\sqrt{x^2 + z^2 + 36}}\mathbf{i} - \frac{6}{\sqrt{x^2 + z^2 + 36}}\mathbf{j} + \frac{z}{\sqrt{x^2 + z^2 + 36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B \mathbf{u}_B = 1610 \left( -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k} \right) = [-460\mathbf{i} - 1380\mathbf{j} + 690\mathbf{k}] \text{ N}$$

$$\begin{aligned} \mathbf{F}_C = F_C \mathbf{u}_C &= 2400 \left( \frac{x}{\sqrt{x^2 + z^2 + 36}}\mathbf{i} - \frac{6}{\sqrt{x^2 + z^2 + 36}}\mathbf{j} + \frac{z}{\sqrt{x^2 + z^2 + 36}}\mathbf{k} \right) \\ &= \frac{2400x}{\sqrt{x^2 + z^2 + 36}}\mathbf{i} - \frac{14400}{\sqrt{x^2 + z^2 + 36}}\mathbf{j} + \frac{2400z}{\sqrt{x^2 + z^2 + 36}}\mathbf{k} \end{aligned}$$

Since the resultant force  $\mathbf{F}_R$  is directed along the negative  $y$  axis, and the load is directed along the  $z$  axis, these two forces can be written as

$$\mathbf{F}_R = -F_R \mathbf{j} \quad \text{and} \quad \mathbf{W} = [-1500\mathbf{k}] \text{ N}$$

**Resultant Force:**

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R \mathbf{j} = (-460\mathbf{i} - 1380\mathbf{j} + 690\mathbf{k}) + \left( \frac{2400x}{\sqrt{x^2 + z^2 + 36}}\mathbf{i} - \frac{14400}{\sqrt{x^2 + z^2 + 36}}\mathbf{j} + \frac{2400z}{\sqrt{x^2 + z^2 + 36}}\mathbf{k} \right) + (-1500\mathbf{k})$$

$$-F_R \mathbf{j} = \left( \frac{2400x}{\sqrt{x^2 + z^2 + 36}} - 460 \right) \mathbf{i} - \left( \frac{14400}{\sqrt{x^2 + z^2 + 36}} + 1380 \right) \mathbf{j} + \left( 690 + \frac{2400z}{\sqrt{x^2 + z^2 + 36}} - 1500 \right) \mathbf{k}$$

Equating the  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  components,

$$0 = \frac{2400x}{\sqrt{x^2 + z^2 + 36}} - 460 \quad \frac{2400x}{\sqrt{x^2 + z^2 + 36}} = 460 \quad (1)$$

$$-F_R = - \left( \frac{14400}{\sqrt{x^2 + z^2 + 36}} + 1380 \right) \quad F_R = \frac{14400}{\sqrt{x^2 + z^2 + 36}} + 1380 \quad (2)$$

$$0 = 690 + \frac{2400z}{\sqrt{x^2 + z^2 + 36}} - 1500 \quad \frac{2400z}{\sqrt{x^2 + z^2 + 36}} = 810 \quad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \quad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

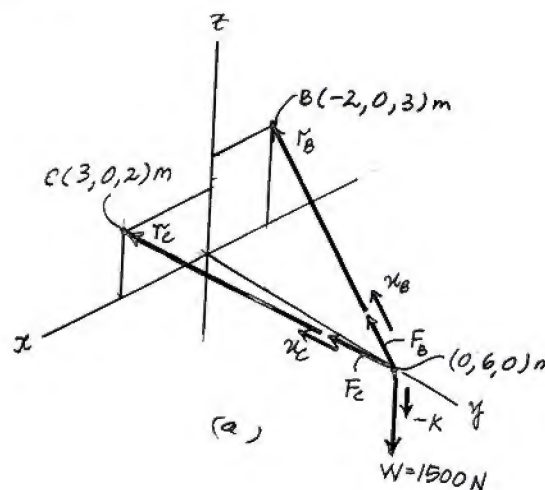
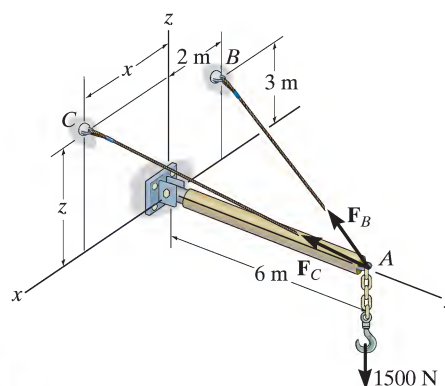
$$z = 2.197 \text{ m} = 2.20 \text{ m} \quad \text{Ans.}$$

Substituting  $z = 2.197$  m into Eq. (4), yields

$$x = 1.248 \text{ m} = 1.25 \text{ m} \quad \text{Ans.}$$

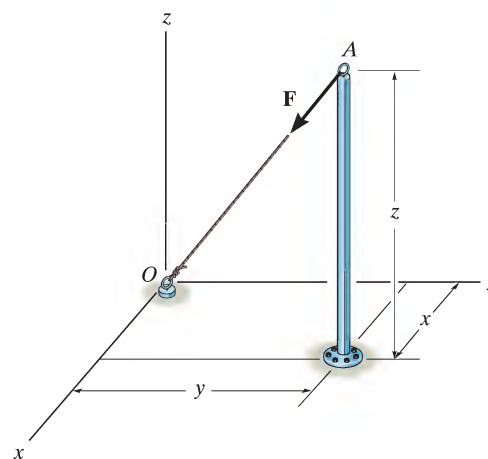
Substituting  $x = 1.248$  m and  $z = 2.197$  m into Eq. (2), yields

$$F_R = 3591.85 \text{ N} = 3.59 \text{ kN} \quad \text{Ans.}$$



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**•2–101.** The cable  $AO$  exerts a force on the top of the pole of  $\mathbf{F} = \{-120\mathbf{i} - 90\mathbf{j} - 80\mathbf{k}\}$  lb. If the cable has a length of 34 ft, determine the height  $z$  of the pole and the location  $(x, y)$  of its base.



$$F = \sqrt{(-120)^2 + (-90)^2 + (-80)^2} = 170 \text{ lb}$$

$$\mathbf{u} = \frac{\mathbf{F}}{F} = -\frac{120}{170}\mathbf{i} - \frac{90}{170}\mathbf{j} - \frac{80}{170}\mathbf{k}$$

$$\mathbf{r} = 34\mathbf{u} = \{-24\mathbf{i} - 18\mathbf{j} - 16\mathbf{k}\} \text{ ft}$$

Thus,

$$x = 24 \text{ ft} \quad \text{Ans}$$

$$y = 18 \text{ ft} \quad \text{Ans}$$

$$z = 16 \text{ ft} \quad \text{Ans}$$

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**2-102.** If the force in each chain has a magnitude of 450 lb, determine the magnitude and coordinate direction angles of the resultant force.

**Force Vectors:** The unit vectors  $\mathbf{u}_A$ ,  $\mathbf{u}_B$ , and  $\mathbf{u}_C$  must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{(-3\sin 30^\circ - 0)\mathbf{i} + (3\cos 30^\circ - 0)\mathbf{j} + (0 - 7)\mathbf{k}}{\sqrt{(-3\sin 30^\circ - 0)^2 + (3\cos 30^\circ - 0)^2 + (0 - 7)^2}} = -0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_B = \frac{(-3\sin 30^\circ - 0)\mathbf{i} + (-3\cos 30^\circ - 0)\mathbf{j} + (0 - 7)\mathbf{k}}{\sqrt{(-3\sin 30^\circ - 0)^2 + (-3\cos 30^\circ - 0)^2 + (0 - 7)^2}} = -0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_C = \frac{(3 - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (0 - 7)\mathbf{k}}{\sqrt{(3 - 0)^2 + (0 - 0)^2 + (0 - 7)^2}} = 0.3939\mathbf{i} - 0.9191\mathbf{k}$$

Thus, the force vectors  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ , and  $\mathbf{F}_C$  are given by

$$\mathbf{F}_A = F_A \mathbf{u}_A = 450(-0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}) = \{-88.63\mathbf{i} + 153.51\mathbf{j} - 413.62\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_B = F_B \mathbf{u}_B = 450(-0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}) = \{-88.63\mathbf{i} - 153.51\mathbf{j} - 413.62\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 450(0.3939\mathbf{i} - 0.9191\mathbf{k}) = \{177.26\mathbf{i} - 413.62\mathbf{k}\} \text{ lb}$$

**Resultant Force:**

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = (-88.63\mathbf{i} + 153.51\mathbf{j} - 413.62\mathbf{k}) + (-88.63\mathbf{i} - 153.51\mathbf{j} - 413.62\mathbf{k}) + (177.26\mathbf{i} - 413.62\mathbf{k}) \\ = \{-1240.85\mathbf{k}\} \text{ lb}$$

The magnitude of  $\mathbf{F}_R$  is

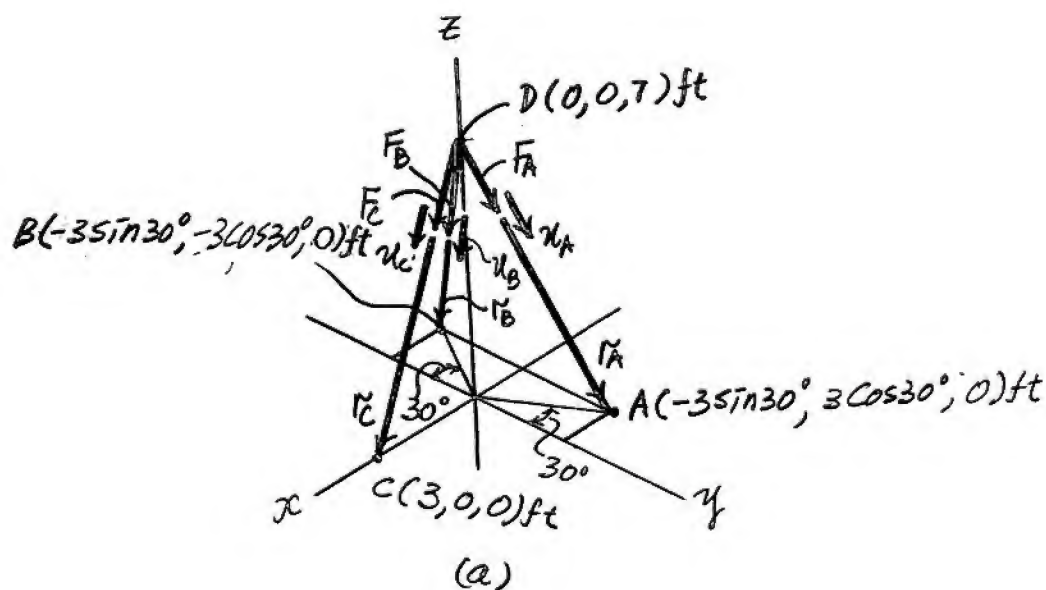
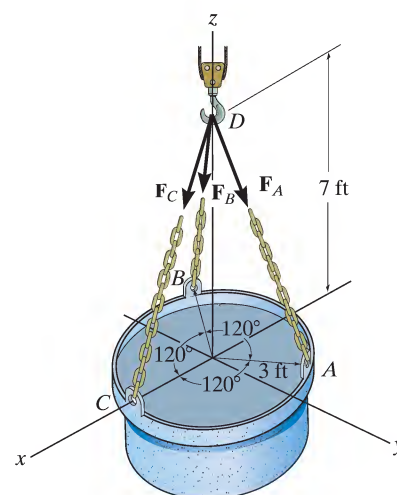
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ = \sqrt{0^2 + 0^2 + (-1240.85)^2} = 1240.85 \text{ lb} = 1.24 \text{ kip} \quad \text{Ans.}$$

The coordinate direction angles of  $\mathbf{F}_R$  are

$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{0}{1240.85} \right) = 90^\circ \quad \text{Ans.}$$

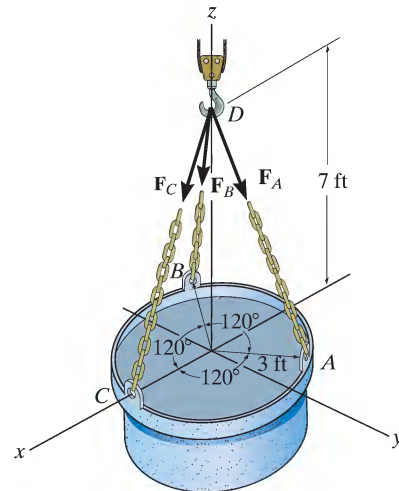
$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{0}{1240.85} \right) = 90^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{-1240.85}{1240.85} \right) = 180^\circ \quad \text{Ans.}$$



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**2-103.** If the resultant of the three forces is  $\mathbf{F}_R = \{-900\mathbf{k}\}$  lb, determine the magnitude of the force in each chain.



**Force Vectors:** The unit vectors  $\mathbf{u}_A$ ,  $\mathbf{u}_B$ , and  $\mathbf{u}_C$  must be determined first. From Fig. a,

$$\mathbf{u}_A = \frac{(-3\sin 30^\circ - 0)\mathbf{i} + (3\cos 30^\circ - 0)\mathbf{j} + (0 - 7)\mathbf{k}}{\sqrt{(-3\sin 30^\circ - 0)^2 + (3\cos 30^\circ - 0)^2 + (0 - 7)^2}} = -0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_B = \frac{(-3\sin 30^\circ - 0)\mathbf{i} + (-3\cos 30^\circ - 0)\mathbf{j} + (0 - 7)\mathbf{k}}{\sqrt{(-3\sin 30^\circ - 0)^2 + (-3\cos 30^\circ - 0)^2 + (0 - 7)^2}} = -0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_C = \frac{(3 - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (0 - 7)\mathbf{k}}{\sqrt{(3 - 0)^2 + (0 - 0)^2 + (0 - 7)^2}} = 0.3939\mathbf{i} - 0.9191\mathbf{k}$$

Thus, the force vectors  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ , and  $\mathbf{F}_C$  are given by

$$\mathbf{F}_A = F_A \mathbf{u}_A = -0.1970F_A\mathbf{i} + 0.3411F_A\mathbf{j} - 0.9191F_A\mathbf{k}$$

$$\mathbf{F}_B = F_B \mathbf{u}_B = -0.1970F_B\mathbf{i} - 0.3411F_B\mathbf{j} - 0.9191F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 0.3939F_C\mathbf{i} - 0.9191F_C\mathbf{k}$$

**Resultant Force:** The vector addition of  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ , and  $\mathbf{F}_C$  is equal to  $\mathbf{F}_R$ . Thus,

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$

$$-900\mathbf{k} = (-0.1970F_A\mathbf{i} + 0.3411F_A\mathbf{j} - 0.9191F_A\mathbf{k}) + (-0.1970F_B\mathbf{i} - 0.3411F_B\mathbf{j} - 0.9191F_B\mathbf{k}) + (0.3939F_C\mathbf{i} - 0.9191F_C\mathbf{k})$$

$$-900\mathbf{k} = (-0.1970F_A - 0.1970F_B + 0.3939F_C)\mathbf{i} + (0.3411F_A - 0.3411F_B)\mathbf{j} + (-0.9191F_A - 0.9191F_B - 0.9191F_C)\mathbf{k}$$

Equating the  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  components,

$$0 = -0.1970F_A - 0.1970F_B + 0.3939F_C \quad (1)$$

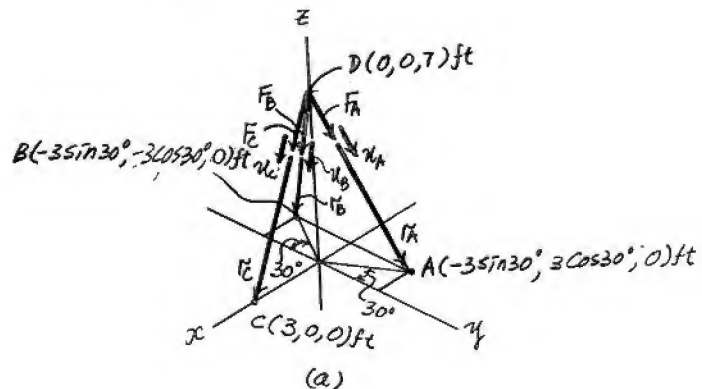
$$0 = 0.3411F_A - 0.3411F_B \quad (2)$$

$$-900\mathbf{k} = -0.9191F_A - 0.9191F_B - 0.9191F_C \quad (3)$$

Solving Eqs. (1), (2), and (3), yields

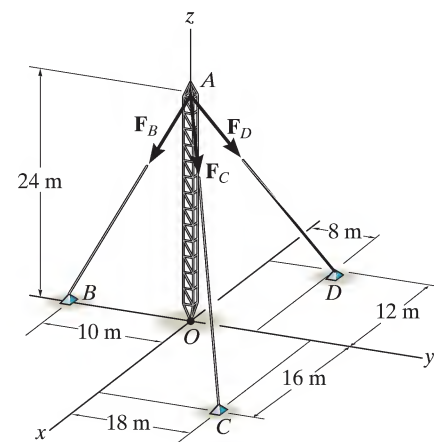
$$F_A = F_B = F_C = 326 \text{ lb}$$

**Ans.**



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**\*2-104.** The antenna tower is supported by three cables. If the forces of these cables acting on the antenna are  $F_B = 520$  N,  $F_C = 680$  N, and  $F_D = 560$  N, determine the magnitude and coordinate direction angles of the resultant force acting at A.



$$\mathbf{F}_B = 520 \left( \frac{\mathbf{r}_{AB}}{r_{AB}} \right) = 520 \left( -\frac{10}{26} \mathbf{j} - \frac{24}{26} \mathbf{k} \right) = -200 \mathbf{j} - 480 \mathbf{k}$$

$$\mathbf{F}_C = 680 \left( \frac{\mathbf{r}_{AC}}{r_{AC}} \right) = 680 \left( \frac{16}{34} \mathbf{i} + \frac{18}{34} \mathbf{j} - \frac{24}{34} \mathbf{k} \right) = 320 \mathbf{i} + 360 \mathbf{j} - 480 \mathbf{k}$$

$$\mathbf{F}_D = 560 \left( \frac{\mathbf{r}_{AD}}{r_{AD}} \right) = 560 \left( -\frac{12}{28} \mathbf{i} + \frac{8}{28} \mathbf{j} - \frac{24}{28} \mathbf{k} \right) = -240 \mathbf{i} + 160 \mathbf{j} - 480 \mathbf{k}$$

$$\mathbf{F}_R = \Sigma \mathbf{F} = (80 \mathbf{i} + 320 \mathbf{j} - 1440 \mathbf{k}) \text{ N}$$

$$F_R = \sqrt{(80)^2 + (320)^2 + (-1440)^2} = 1477.3 = 1.48 \text{ kN} \quad \text{Ans}$$

$$\alpha = \cos^{-1} \left( \frac{80}{1477.3} \right) = 86.9^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1} \left( \frac{320}{1477.3} \right) = 77.5^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1} \left( \frac{-1440}{1477.3} \right) = 167^\circ \quad \text{Ans}$$

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•2–105. If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.

**Force Vectors:** The unit vectors  $\mathbf{u}_A$ ,  $\mathbf{u}_B$ ,  $\mathbf{u}_C$  and  $\mathbf{u}_D$  of  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ ,  $\mathbf{F}_C$  and  $\mathbf{F}_D$  must be determined first. From Fig. a,

$$\begin{aligned}\mathbf{u}_A &= \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \\ \mathbf{u}_B &= \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \\ \mathbf{u}_C &= \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \\ \mathbf{u}_D &= \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\end{aligned}$$

Thus, the force vectors  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ ,  $\mathbf{F}_C$  and  $\mathbf{F}_D$  are given by

$$\begin{aligned}\mathbf{F}_A &= F_A \mathbf{u}_A = 70 \left( \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = [30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}] \text{ lb} \\ \mathbf{F}_B &= F_B \mathbf{u}_B = 70 \left( \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = [30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}] \text{ lb} \\ \mathbf{F}_C &= F_C \mathbf{u}_C = 70 \left( -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = [-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}] \text{ lb} \\ \mathbf{F}_D &= F_D \mathbf{u}_D = 70 \left( -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = [-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}] \text{ lb}\end{aligned}$$

**Resultant Force:**

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) + (30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) \\ &= [-240\mathbf{k}] \text{ N}\end{aligned}$$

The magnitude of  $\mathbf{F}_R$  is

$$\begin{aligned}F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{0 + 0 + (-240)^2} = 240 \text{ lb}\end{aligned}$$

Ans.

The coordinate direction angles of  $\mathbf{F}_R$  are

$$\alpha = \cos^{-1} \left[ \frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left( \frac{0}{240} \right) = 90^\circ$$

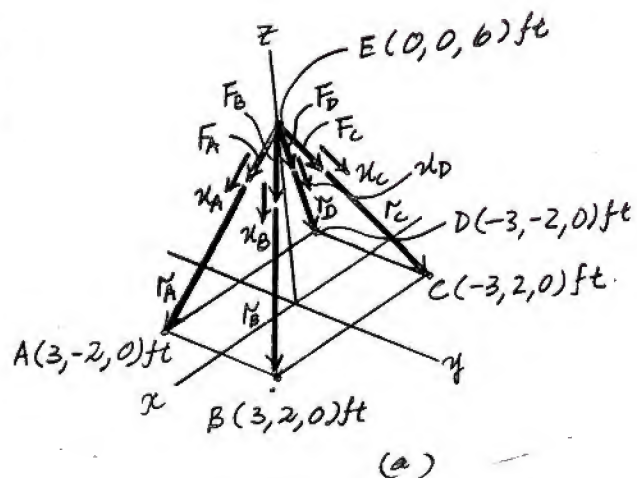
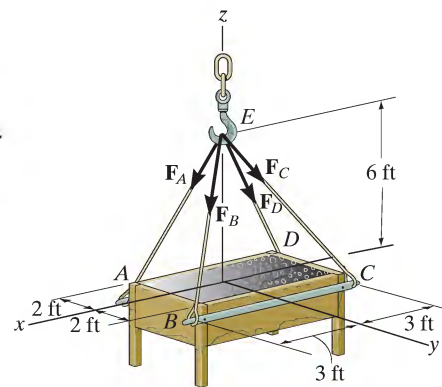
Ans.

$$\beta = \cos^{-1} \left[ \frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left( \frac{0}{240} \right) = 90^\circ$$

Ans.

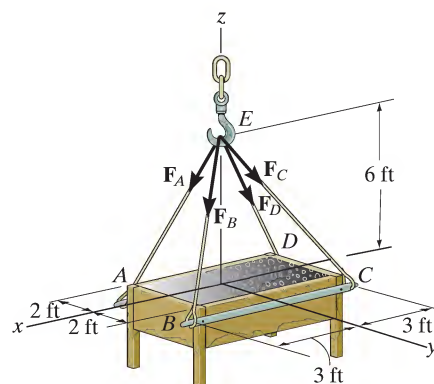
$$\gamma = \cos^{-1} \left[ \frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left( \frac{-240}{240} \right) = 180^\circ$$

Ans.



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**2-106.** If the resultant of the four forces is  $\mathbf{F}_R = \{-360\mathbf{k}\}$  lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



**Force Vectors:** The unit vectors  $\mathbf{u}_A$ ,  $\mathbf{u}_B$ ,  $\mathbf{u}_C$  and  $\mathbf{u}_D$  of  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ ,  $\mathbf{F}_C$  and  $\mathbf{F}_D$  must be determined first.

From Fig. a,

$$\begin{aligned}\mathbf{u}_A &= \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \\ \mathbf{u}_B &= \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \\ \mathbf{u}_C &= \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \\ \mathbf{u}_D &= \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\end{aligned}$$

Since the magnitudes of  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ ,  $\mathbf{F}_C$  and  $\mathbf{F}_D$  are the same and denoted as  $F$ , they can be written as

$$\begin{aligned}\mathbf{F}_A &= F\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) \\ \mathbf{F}_B &= F\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) \\ \mathbf{F}_C &= F\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) \\ \mathbf{F}_D &= F\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\end{aligned}$$

**Resultant Force:** The vector addition of  $\mathbf{F}_A$ ,  $\mathbf{F}_B$ ,  $\mathbf{F}_C$  and  $\mathbf{F}_D$  is equal to  $\mathbf{F}_R$ . Thus,

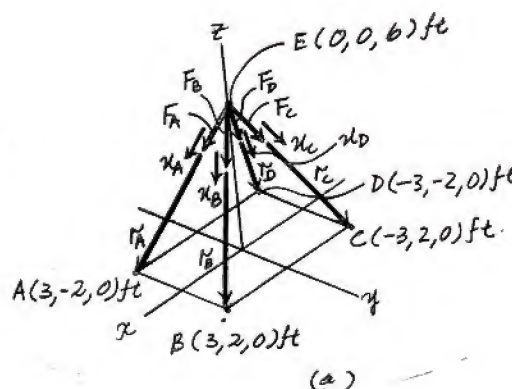
$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D \\ \{-360\mathbf{k}\} &= \left[F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] \\ -360\mathbf{k} &= -\frac{24}{7}\mathbf{k} F\end{aligned}$$

Thus,

$$360 = \frac{24}{7} F$$

$$F = 105 \text{ lb}$$

Ans.



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**2-107.** The pipe is supported at its end by a cord  $AB$ . If the cord exerts a force of  $F = 12$  lb on the pipe at  $A$ , express this force as a Cartesian vector.

**Unit Vector:** The coordinates of point  $A$  are

$$A(5, 3\cos 20^\circ, -3\sin 20^\circ) \text{ ft} = A(5.00, 2.819, -1.026) \text{ ft}$$

Then

$$\begin{aligned} \mathbf{r}_{AB} &= \{(0 - 5.00)\mathbf{i} + (0 - 2.819)\mathbf{j} + [6 - (-1.026)]\mathbf{k}\} \text{ ft} \\ &= \{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}\} \text{ ft} \end{aligned}$$

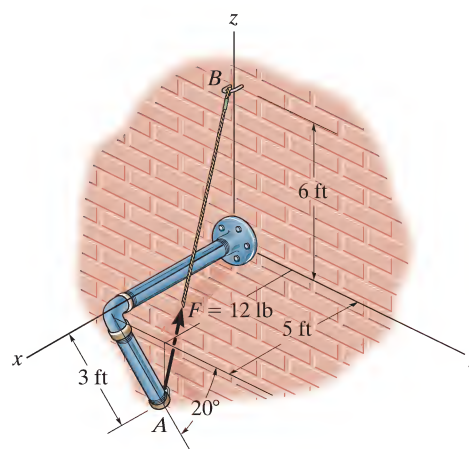
$$r_{AB} = \sqrt{(-5.00)^2 + (-2.819)^2 + 7.026^2} = 9.073 \text{ ft}$$

$$\begin{aligned} \mathbf{u}_{AB} &= \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}}{9.073} \\ &= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k} \end{aligned}$$

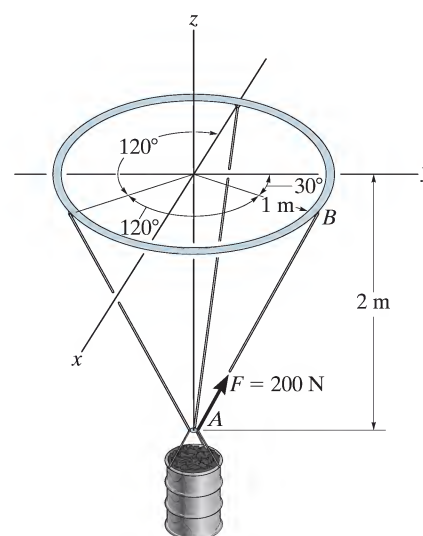
**Force Vector:**

$$\begin{aligned} \mathbf{F} &= F\mathbf{u}_{AB} = 12\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\} \text{ lb} \\ &= \{-6.61\mathbf{i} - 3.73\mathbf{j} + 9.29\mathbf{k}\} \text{ lb} \end{aligned}$$

**Ans**



**\*2-108.** The load at  $A$  creates a force of 200 N in wire  $AB$ . Express this force as a Cartesian vector, acting on  $A$  and directed towards  $B$ .



$$\begin{aligned} \mathbf{r}_{AB} &= (1\sin 30^\circ - 0)\mathbf{i} + (1\cos 30^\circ - 0)\mathbf{j} + (2 - 0)\mathbf{k} \\ &= (0.5\mathbf{i} + 0.866\mathbf{j} + 2\mathbf{k}) \text{ m} \end{aligned}$$

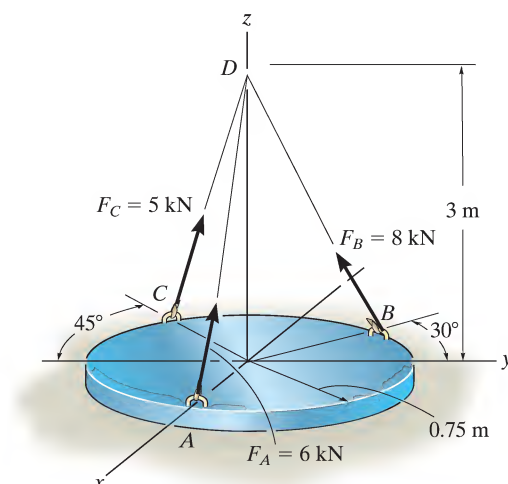
$$r_{AB} = \sqrt{(0.5)^2 + (0.866)^2 + (2)^2} = 2.236 \text{ m}$$

$$\mathbf{u}_{AB} = \left( \frac{\mathbf{r}_{AB}}{r_{AB}} \right) = 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

$$\mathbf{F} = 200\mathbf{u}_{AB} = \{44.7\mathbf{i} + 77.5\mathbf{j} + 179\mathbf{k}\} \text{ N} \quad \mathbf{Ans}$$

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•2–109. The cylindrical plate is subjected to the three cable forces which are concurrent at point  $D$ . Express each force which the cables exert on the plate as a Cartesian vector, and determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{r}_A = (0 - 0.75)\mathbf{i} + (0 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-0.75\mathbf{i} + 3\mathbf{k}\} \text{ m}$$

$$r_A = \sqrt{(-0.75)^2 + 0^2 + 3^2} = 3.0923 \text{ m}$$

$$\mathbf{F}_A = F_A \left( \frac{\mathbf{r}_A}{r_A} \right) = 6 \left( \frac{-0.75\mathbf{i} + 3\mathbf{k}}{3.0923} \right)$$

$$= \{-1.4552\mathbf{i} + 5.8209\mathbf{k}\} \text{ kN}$$

$$= \{-1.46\mathbf{i} + 5.82\mathbf{k}\} \text{ kN}$$

Ans

$$\mathbf{r}_C = [0 - (-0.75 \sin 45^\circ)]\mathbf{i} + [0 - (0.75 \cos 45^\circ)]\mathbf{j} + (3 - 0)\mathbf{k}$$

$$= \{0.5303\mathbf{i} + 0.5303\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_C = \sqrt{(0.5303)^2 + (0.5303)^2 + 3^2} = 3.0923 \text{ m}$$

$$\mathbf{F}_C = F_C \left( \frac{\mathbf{r}_C}{r_C} \right) = 5 \left( \frac{0.5303\mathbf{i} + 0.5303\mathbf{j} + 3\mathbf{k}}{3.0923} \right)$$

$$= \{0.8575\mathbf{i} + 0.8575\mathbf{j} + 4.8507\mathbf{k}\} \text{ kN}$$

$$= \{0.857\mathbf{i} + 0.857\mathbf{j} + 4.85\mathbf{k}\} \text{ kN}$$

Ans

$$\mathbf{r}_B = [0 - (-0.75 \sin 30^\circ)]\mathbf{i} + [0 - (0.75 \cos 30^\circ)]\mathbf{j} + (3 - 0)\mathbf{k}$$

$$= \{0.375\mathbf{i} - 0.6495\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_B = \sqrt{(0.375)^2 + (-0.6495)^2 + 3^2} = 3.0923 \text{ m}$$

$$\mathbf{F}_B = F_B \left( \frac{\mathbf{r}_B}{r_B} \right) = 8 \left( \frac{0.375\mathbf{i} - 0.6495\mathbf{j} + 3\mathbf{k}}{3.0923} \right)$$

$$= \{0.9701\mathbf{i} - 1.6803\mathbf{j} + 7.7611\mathbf{k}\} \text{ kN}$$

$$= \{0.970\mathbf{i} - 1.68\mathbf{j} + 7.76\mathbf{k}\} \text{ kN}$$

Ans

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$

$$= \{-1.4552\mathbf{i} + 5.8209\mathbf{k}\} + \{0.9701\mathbf{i} - 1.6803\mathbf{j} + 7.7611\mathbf{k}\}$$

$$+ \{0.8575\mathbf{i} + 0.8575\mathbf{j} + 4.8507\mathbf{k}\}$$

$$= \{0.3724\mathbf{i} - 0.8228\mathbf{j} + 18.4327\mathbf{k}\} \text{ kN}$$

$$F_R = \sqrt{(0.3724)^2 + (-0.8228)^2 + (18.4327)^2}$$

$$= 18.4548 \text{ kN} = 18.5 \text{ kN}$$

Ans

$$u_R = \frac{\mathbf{F}_R}{F_R} = \frac{0.3724\mathbf{i} - 0.8228\mathbf{j} + 18.4327\mathbf{k}}{18.4548}$$

$$= 0.02018\mathbf{i} - 0.04459\mathbf{j} + 0.9988\mathbf{k}$$

$$\cos \alpha = 0.02018$$

$$\alpha = 88.8^\circ$$

Ans

$$\cos \beta = -0.04458$$

$$\beta = 92.6^\circ$$

Ans

$$\cos \gamma = 0.9988$$

$$\gamma = 2.81^\circ$$

Ans

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**2–110.** The cable attached to the shear-leg derrick exerts a force on the derrick of  $F = 350$  lb. Express this force as a Cartesian vector.

**Unit Vector :** The coordinates of point  $B$  are

$$B(50\sin 30^\circ, 50\cos 30^\circ, 0) \text{ ft} = B(25.0, 43.301, 0) \text{ ft}$$

Then

$$\begin{aligned} \mathbf{r}_{AB} &= \{(25.0 - 0)\mathbf{i} + (43.301 - 0)\mathbf{j} + (0 - 35)\mathbf{k}\} \text{ ft} \\ &= \{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}\} \text{ ft} \end{aligned}$$

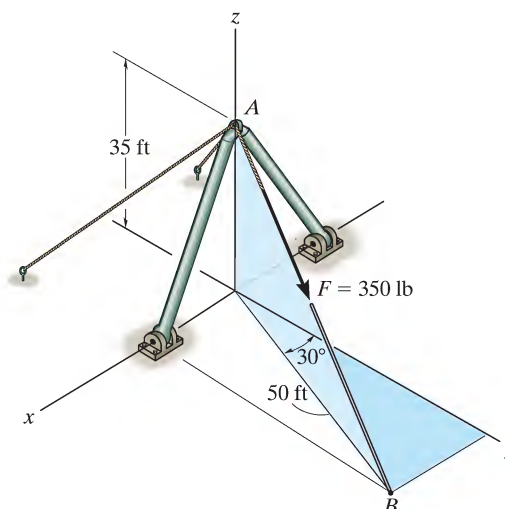
$$r_{AB} = \sqrt{25.0^2 + 43.301^2 + (-35.0)^2} = 61.033 \text{ ft}$$

$$\begin{aligned} \mathbf{u}_{AB} &= \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}}{61.033} \\ &= 0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k} \end{aligned}$$

**Force Vector :**

$$\begin{aligned} \mathbf{F} &= F\mathbf{u}_{AB} = 350\{0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}\} \text{ lb} \\ &= \{143\mathbf{i} + 248\mathbf{j} - 201\mathbf{k}\} \text{ lb} \end{aligned}$$

**Ans**



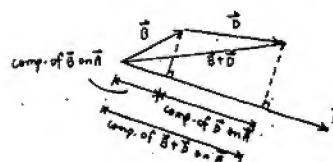
**2–111.** Given the three vectors  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{D}$ , show that  $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$ .

Since the component of  $(\mathbf{B} + \mathbf{D})$  is equal to the sum of the components of  $\mathbf{B}$  and  $\mathbf{D}$ , then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D} \quad (\text{QED})$$

Also,

$$\begin{aligned} \mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot [(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}] \\ &= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z) \\ &= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z) \\ &= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D}) \quad (\text{QED}) \end{aligned}$$



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**\*2-112.** Determine the projected component of the force  $F_{AB} = 560$  N acting along cable AC. Express the result as a Cartesian vector.

**Force Vectors:** The unit vectors  $\mathbf{u}_{AB}$  and  $\mathbf{u}_{AC}$  must be determined first. From Fig. a,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(-1.5-0)\mathbf{i} + (0-3)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-1.5-0)^2 + (0-3)^2 + (1-0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(1.5-0)\mathbf{i} + (0-3)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(1.5-0)^2 + (0-3)^2 + (3-0)^2}} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Thus, the force vector  $\mathbf{F}_{AB}$  is given by

$$\mathbf{F}_{AB} = F_{AB} \mathbf{u}_{AB} = 560 \left( -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k} \right) = [-240\mathbf{i} - 480\mathbf{j} + 160\mathbf{k}] \text{ N}$$

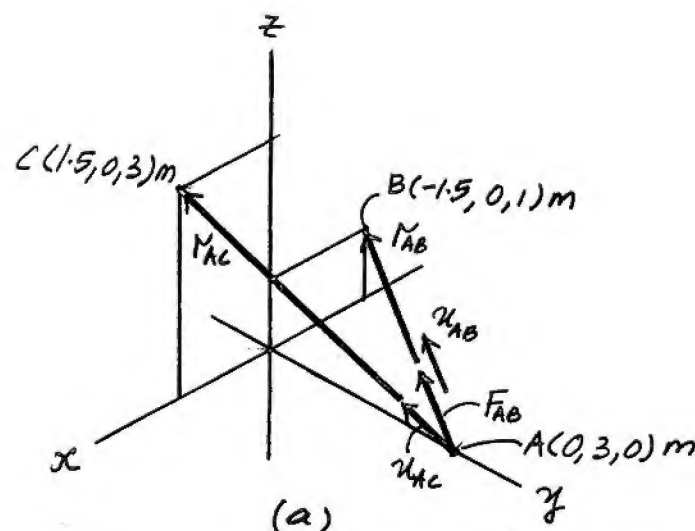
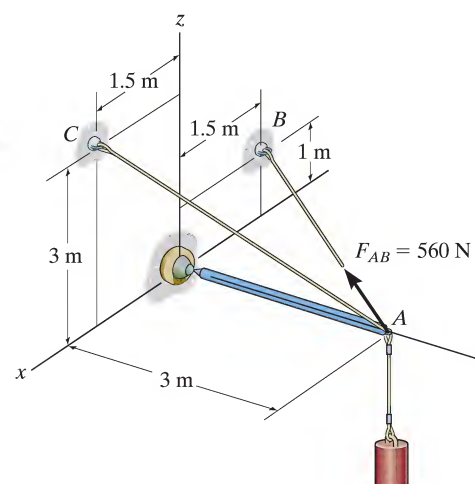
**Vector Dot Product:** The magnitude of the projected component of  $\mathbf{F}_{AB}$  is

$$\begin{aligned} (F_{AB})_{AC} &= \mathbf{F}_{AB} \cdot \mathbf{u}_{AC} = (-240\mathbf{i} - 480\mathbf{j} + 160\mathbf{k}) \cdot \left( \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k} \right) \\ &= (-240) \left( \frac{1}{3} \right) + (-480) \left( -\frac{2}{3} \right) + 160 \left( \frac{2}{3} \right) \\ &= 346.67 \text{ N} \end{aligned}$$

Thus,  $(\mathbf{F}_{AB})_{AC}$  expressed in Cartesian vector form is

$$\begin{aligned} (\mathbf{F}_{AB})_{AC} &= (F_{AB})_{AC} \mathbf{u}_{AC} = 346.67 \left( \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k} \right) \\ &= [116\mathbf{i} - 231\mathbf{j} + 231\mathbf{k}] \text{ N} \end{aligned}$$

**Ans.**



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•2–113. Determine the magnitudes of the components of force  $F = 56$  N acting along and perpendicular to line  $AO$ .

**Unit Vectors:** The unit vectors  $\mathbf{u}_{AD}$  and  $\mathbf{u}_{AO}$  must be determined first. From Fig. *a*,

$$\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{r_{AD}} = \frac{[0 - (-1.5)]\mathbf{i} + (0 - 3)\mathbf{j} + (2 - 1)\mathbf{k}}{\sqrt{[0 - (-1.5)]^2 + (0 - 3)^2 + (2 - 1)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{[0 - (-1.5)]\mathbf{i} + (0 - 3)\mathbf{j} + (0 - 1)\mathbf{k}}{\sqrt{[0 - (-1.5)]^2 + (0 - 3)^2 + (0 - 1)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$$

Thus, the force vector  $\mathbf{F}$  is given by

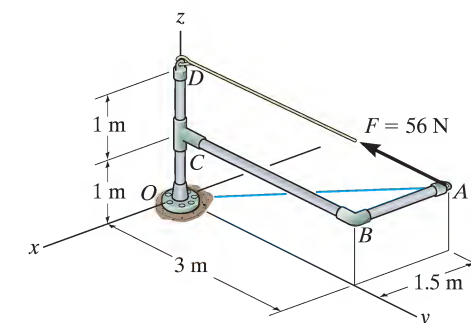
$$\mathbf{F} = F\mathbf{u}_{AD} = 56\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = [24\mathbf{i} - 48\mathbf{j} + 16\mathbf{k}]\text{N}$$

**Vector Dot Product:** The magnitude of the projected component of  $\mathbf{F}$  parallel to line  $AO$  is

$$(F_{AO})_{\text{paral}} = \mathbf{F} \cdot \mathbf{u}_{AO} = (24\mathbf{i} - 48\mathbf{j} + 16\mathbf{k}) \cdot \left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}\right)$$

$$= (24)\left(\frac{3}{7}\right) + (-48)\left(-\frac{6}{7}\right) + (16)\left(-\frac{2}{7}\right)$$

$$= 46.86 \text{ N} = 46.9 \text{ N}$$



Ans.

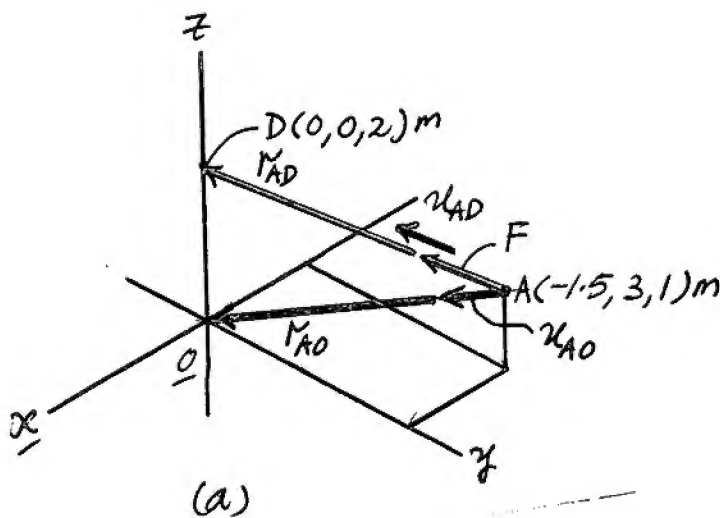
The component of  $\mathbf{F}$  perpendicular to line  $AO$  is

$$(F_{AO})_{\text{per}} = \sqrt{F^2 - (F_{AO})_{\text{paral}}^2}$$

$$= \sqrt{56^2 - 46.86^2}$$

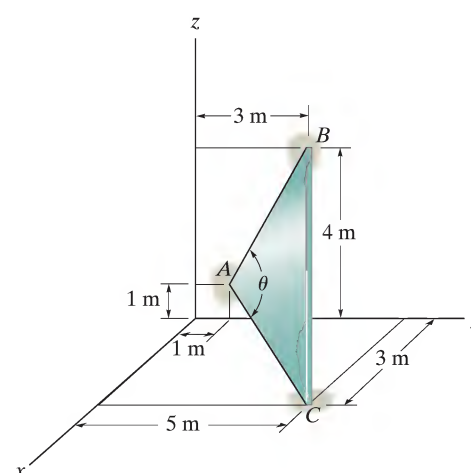
$$= 30.7 \text{ N}$$

Ans.



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**2–114.** Determine the length of side  $BC$  of the triangular plate. Solve the problem by finding the magnitude of  $\mathbf{r}_{BC}$ ; then check the result by first finding  $\theta$ ,  $r_{AB}$ , and  $r_{AC}$  and then using the cosine law.



$$\mathbf{r}_{BC} = \{3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\} \text{ m}$$

$$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39 \text{ m} \quad \text{Ans}$$

Also,

$$\mathbf{r}_{AC} = \{3\mathbf{i} + 4\mathbf{j} - 1\mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990 \text{ m}$$

$$\mathbf{r}_{AB} = \{2\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056 \text{ m}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

$$\theta = \cos^{-1} \left( \frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}} \right) = \cos^{-1} \frac{5}{(5.0990)(3.6056)}$$

$$\theta = 74.219^\circ$$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056) \cos 74.219^\circ}$$

$$r_{BC} = 5.39 \text{ m} \quad \text{Ans}$$

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**2-115.** Determine the magnitudes of the components of  $F = 600$  N acting along and perpendicular to segment  $DE$  of the pipe assembly.

**Unit Vectors:** The unit vectors  $\mathbf{u}_{EB}$  and  $\mathbf{u}_{ED}$  must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector  $\mathbf{F}$  is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}] \text{ N}$$

**Vector Dot Product:** The magnitude of the component of  $\mathbf{F}$  parallel to segment  $DE$  of the pipe assembly is

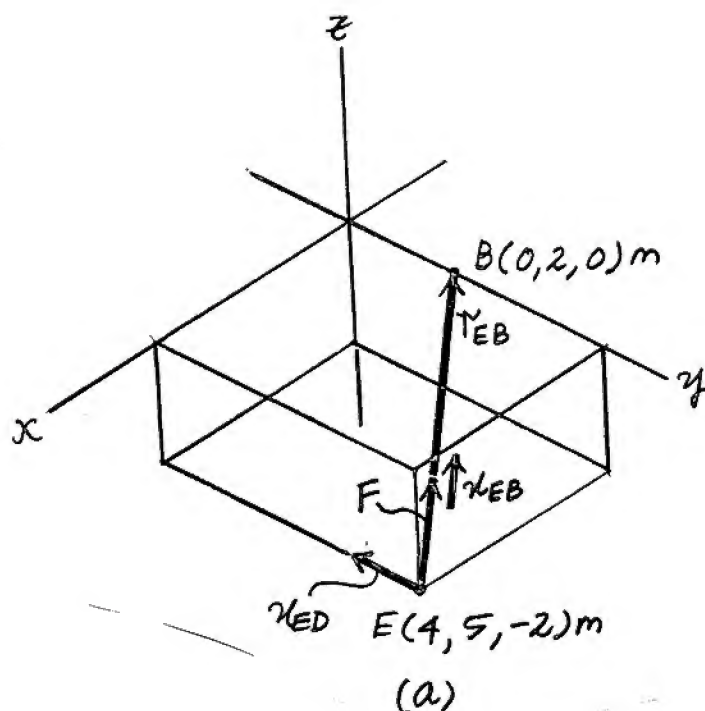
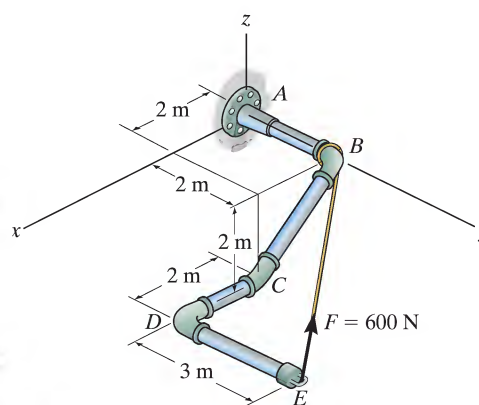
$$\begin{aligned} (F_{ED})_{\text{paral}} &= \mathbf{F} \cdot \mathbf{u}_{ED} = (-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}) \cdot (-\mathbf{j}) \\ &= (-445.66)(0) + (-334.25)(-1) + (222.83)(0) \\ &= 334.25 = 334 \text{ N} \end{aligned}$$

**Ans.**

The component of  $\mathbf{F}$  perpendicular to segment  $DE$  of the pipe assembly is

$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}^2} = \sqrt{600^2 - 334.25^2} = 498 \text{ N}$$

**Ans.**



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**\*2-116.** Two forces act on the hook. Determine the angle  $\theta$  between them. Also, what are the projections of  $\mathbf{F}_1$  and  $\mathbf{F}_2$  along the  $y$  axis?

$$\mathbf{F}_1 = 600 \cos 120^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 45^\circ \mathbf{k}$$

$$= -300\mathbf{i} + 300\mathbf{j} + 424.3\mathbf{k}; F_1 = 600 \text{ N}$$

$$\mathbf{F}_2 = 120\mathbf{i} + 90\mathbf{j} - 80\mathbf{k}; F_2 = 170 \text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

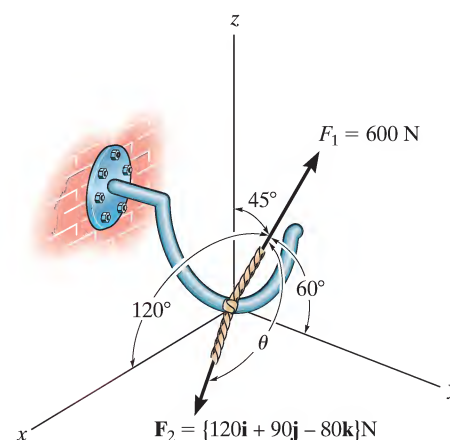
$$\theta = \cos^{-1} \left( \frac{-42\,944}{(170)(600)} \right) \approx 115^\circ \quad \text{Ans}$$

$$\mathbf{u} = \mathbf{j}$$

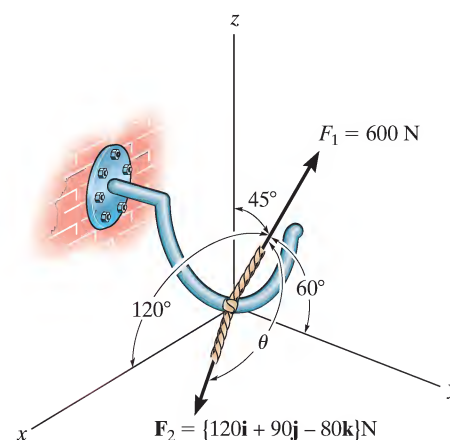
So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300 \text{ N} \quad \text{Ans}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90 \text{ N} \quad \text{Ans}$$



**•2-117.** Two forces act on the hook. Determine the magnitude of the projection of  $\mathbf{F}_2$  along  $\mathbf{F}_1$ .



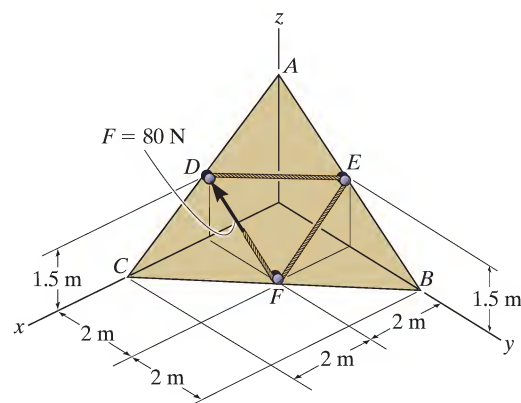
$$\mathbf{u}_1 = \cos 120^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 45^\circ \mathbf{k}$$

$$\text{Proj } \mathbf{F}_2 = \mathbf{F}_2 \cdot \mathbf{u}_1 = (120)(\cos 120^\circ) + (90)(\cos 60^\circ) + (-80)(\cos 45^\circ)$$

$$|\text{Proj } \mathbf{F}_2| = 71.6 \text{ N} \quad \text{Ans}$$

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**2-118.** Determine the projection of force  $F = 80$  N along line  $BC$ . Express the result as a Cartesian vector.



**Unit Vectors:** The unit vectors  $\mathbf{u}_{FD}$  and  $\mathbf{u}_{FC}$  must be determined first. From Fig.  $a$ ,

$$\mathbf{u}_{FD} = \frac{\mathbf{r}_{FD}}{r_{FD}} = \frac{(2-2)\mathbf{i} + (0-2)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(2-2)^2 + (0-2)^2 + (1.5-0)^2}} = -\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

$$\mathbf{u}_{FC} = \frac{\mathbf{r}_{FC}}{r_{FC}} = \frac{(4-2)\mathbf{i} + (0-2)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(4-2)^2 + (0-2)^2 + (0-0)^2}} = 0.7071\mathbf{i} - 0.7071\mathbf{j}$$

Thus, the force vector  $\mathbf{F}$  is given by

$$\mathbf{F} = F\mathbf{u}_{FD} = 80\left(-\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = [-64\mathbf{j} + 48\mathbf{k}] \text{ N}$$

**Vector Dot Product:** The magnitude of the projected component of  $\mathbf{F}$  along line  $BC$  is

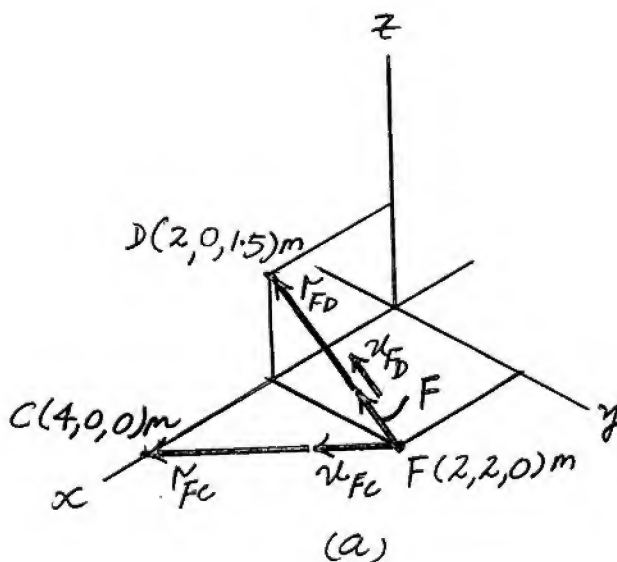
$$\begin{aligned} F_{BC} &= \mathbf{F} \cdot \mathbf{u}_{FC} = (-64\mathbf{j} + 48\mathbf{k}) \cdot (0.7071\mathbf{i} - 0.7071\mathbf{j}) \\ &= (0)(0.7071) + (-64)(-0.7071) + 48(0) \\ &= 45.25 = 45.2 \text{ N} \end{aligned}$$

**Ans.**

The component of  $\mathbf{F}_{BC}$  can be expressed in Cartesian vector form as

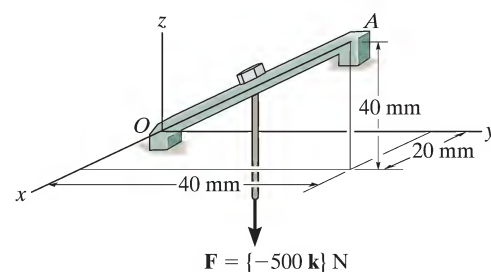
$$\begin{aligned} \mathbf{F}_{BC} &= F_{BC}(\mathbf{u}_{FC}) = 45.25(0.7071\mathbf{i} - 0.7071\mathbf{j}) \\ &= \{32\mathbf{i} - 32\mathbf{j}\} \text{ N} \end{aligned}$$

**Ans.**



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**2–119.** The clamp is used on a jig. If the vertical force acting on the bolt is  $\mathbf{F} = \{-500\mathbf{k}\}$  N, determine the magnitudes of its components  $F_1$  and  $F_2$  which act along the  $OA$  axis and perpendicular to it.



**Unit Vector :** The unit vector along  $OA$  axis is

$$\mathbf{u}_{AO} = \frac{(0-20)\mathbf{i} + (0-40)\mathbf{j} + (0-40)\mathbf{k}}{\sqrt{(0-20)^2 + (0-40)^2 + (0-40)^2}} = -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

**Projected Component of  $\mathbf{F}$  Along  $OA$  Axis :**

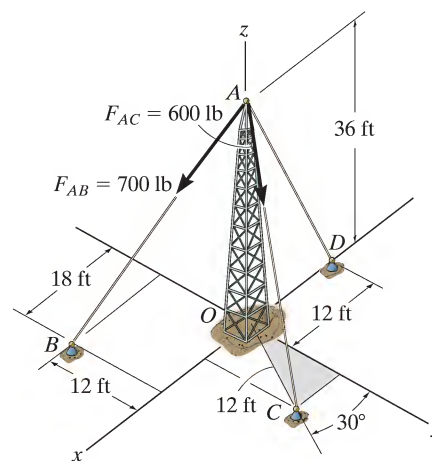
$$\begin{aligned} F_1 &= \mathbf{F} \cdot \mathbf{u}_{AO} = (-500\mathbf{k}) \cdot \left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) \\ &= (0)\left(-\frac{1}{3}\right) + (0)\left(-\frac{2}{3}\right) + (-500)\left(-\frac{2}{3}\right) \\ &= 333.33 \text{ N} = 333 \text{ N} \end{aligned} \quad \text{Ans}$$

**Component of  $\mathbf{F}$  Perpendicular to  $OA$  Axis :** Since the magnitude of force  $\mathbf{F}$  is  $F = 500$  N so that

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{500^2 - 333.33^2} = 373 \text{ N} \quad \text{Ans}$$

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**\*2-120.** Determine the magnitude of the projected component of force  $\mathbf{F}_{AB}$  acting along the  $z$  axis.



**Unit Vector:** The unit vector  $\mathbf{u}_{AB}$  must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(18-0)\mathbf{i} + (-12-0)\mathbf{j} + (0-36)\mathbf{k}}{\sqrt{(18-0)^2 + (-12-0)^2 + (0-36)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector  $\mathbf{F}_{AB}$  is given by

$$\mathbf{F}_{AB} = F_{AB} \mathbf{u}_{AB} = 700 \left( \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}\} \text{ lb}$$

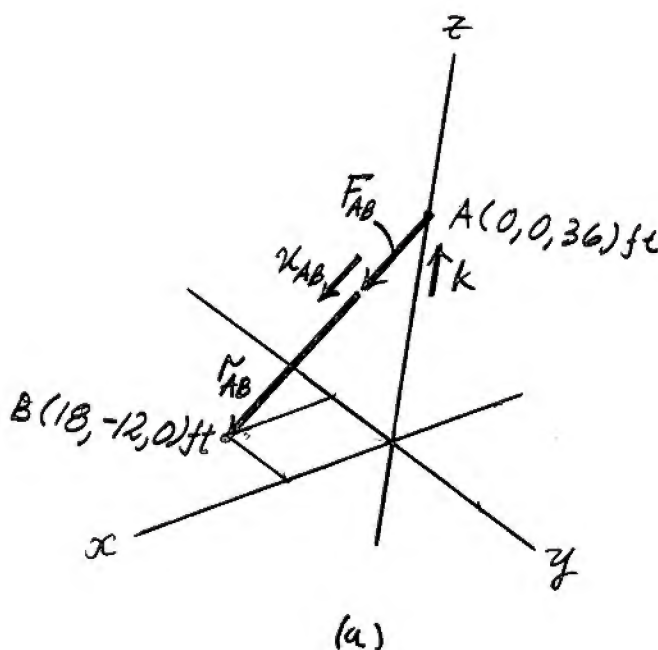
**Vector Dot Product:** The projected component of  $\mathbf{F}_{AB}$  along the  $z$  axis is

$$\begin{aligned} (F_{AB})_z &= \mathbf{F}_{AB} \cdot \mathbf{k} = (300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}) \cdot \mathbf{k} \\ &= -600 \text{ lb} \end{aligned}$$

The negative sign indicates that  $(F_{AB})_z$  is directed towards the negative  $z$  axis. Thus

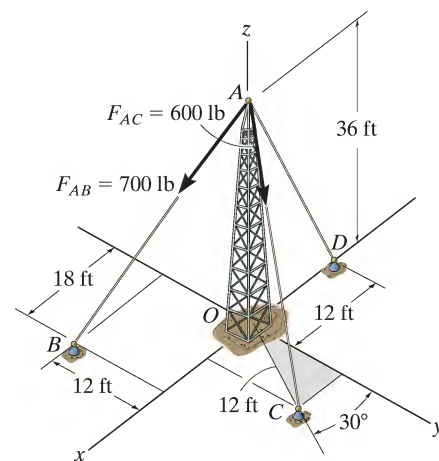
$$(F_{AB})_z = 600 \text{ lb}$$

**Ans.**



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•2–121. Determine the magnitude of the projected component of force  $\mathbf{F}_{AC}$  acting along the  $z$  axis.



**Unit Vector:** The unit vector  $\mathbf{u}_{AC}$  must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(12 \sin 30^\circ - 0)\mathbf{i} + (12 \cos 30^\circ - 0)\mathbf{j} + (0 - 36)\mathbf{k}}{\sqrt{(12 \sin 30^\circ - 0)^2 + (12 \cos 30^\circ - 0)^2 + (0 - 36)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector  $\mathbf{F}_{AC}$  is given by

$$\mathbf{F}_{AC} = F_{AC} \mathbf{u}_{AC} = 600(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}) = \{94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}\} \text{ N}$$

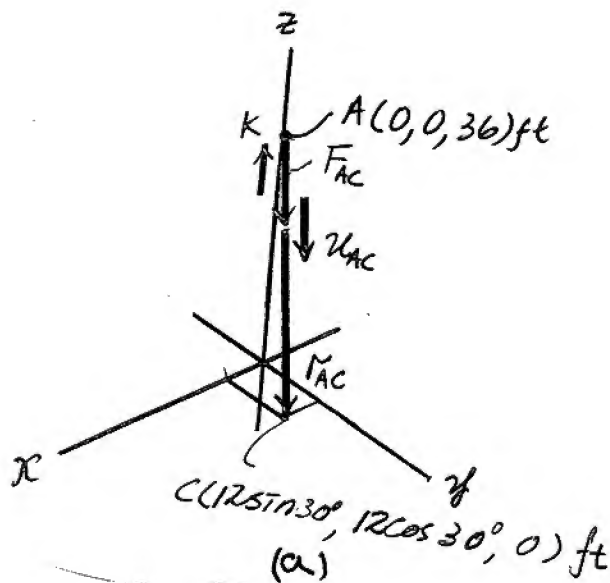
**Vector Dot Product:** The projected component of  $\mathbf{F}_{AC}$  along the  $z$  axis is

$$(F_{AC})_z = \mathbf{F}_{AC} \cdot \mathbf{k} = (94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}) \cdot \mathbf{k} = -569 \text{ lb}$$

The negative sign indicates that  $(F_{AC})_z$  is directed towards the negative  $z$  axis. Thus

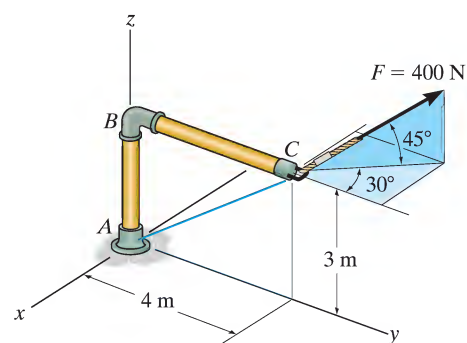
$$(F_{AC})_z = 569 \text{ lb}$$

Ans.



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**2-122.** Determine the projection of force  $F = 400$  N acting along line  $AC$  of the pipe assembly. Express the result as a Cartesian vector.



**Force and unit Vector:** The force vector  $\mathbf{F}$  and unit vector  $\mathbf{u}_{AC}$  must be determined first.

From Fig. (a)

$$\begin{aligned}\mathbf{F} &= 400(-\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}) \\ &= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\} \\ \mathbf{u}_{AC} &= \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\end{aligned}$$

**Vector Dot Product:** The magnitude of the projected component of  $\mathbf{F}$  along line  $AC$  is

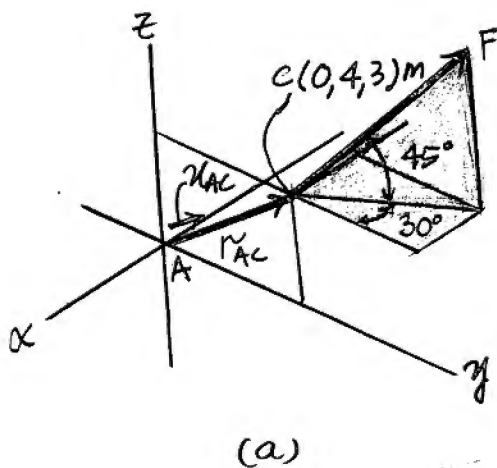
$$\begin{aligned}F_{AC} &= \mathbf{F} \cdot \mathbf{u}_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) \\ &= (-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right) \\ &= 365.66 \text{ lb}\end{aligned}$$

**Ans.**

Thus,  $F_{AC}$  written in Cartesian vector form is

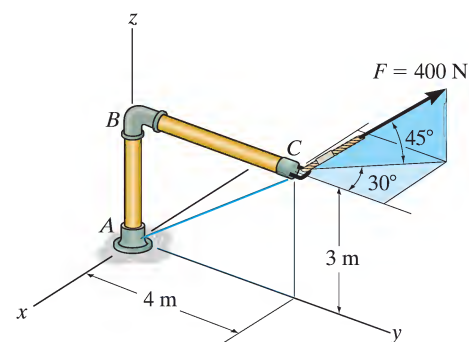
$$\mathbf{F}_{AC} = F_{AC} \mathbf{u}_{AC} = 365.66 \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\} \text{ lb}$$

**Ans.**



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**2-123.** Determine the magnitudes of the components of force  $F = 400$  N acting parallel and perpendicular to segment  $BC$  of the pipe assembly.



**Force Vector:** The force vector  $\mathbf{F}$  must be determined first. From Fig.  $a$ ,

$$\begin{aligned}\mathbf{F} &= 400(-\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}) \\ &= \{-141.42 \mathbf{i} + 244.95 \mathbf{j} + 282.84 \mathbf{k}\} \text{ N}\end{aligned}$$

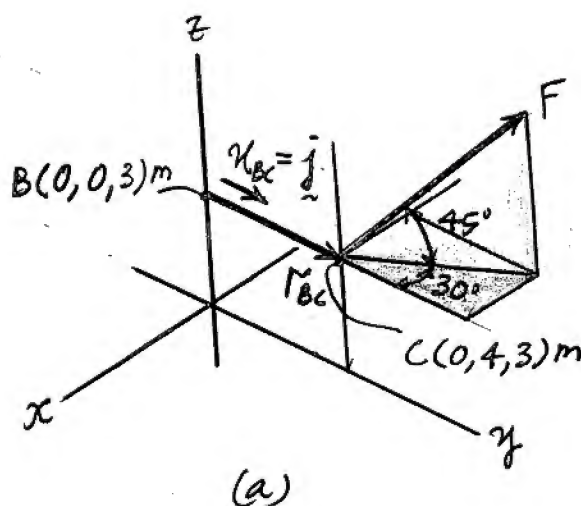
**Vector Dot Product:** By inspecting Fig.  $(a)$  we notice that  $u_{BC} = \mathbf{j}$ . Thus, the magnitude of the component of  $\mathbf{F}$  parallel to segment  $BC$  of the pipe assembly is

$$\begin{aligned}(F_{BC})_{\text{para}} &= \mathbf{F} \cdot \mathbf{j} = (-141.42 \mathbf{i} + 244.95 \mathbf{j} + 282.84 \mathbf{k}) \cdot \mathbf{j} \\ &= -141.42(0) + 244.95(1) + 282.84(0) \\ &= 244.95 \text{ lb} = 245 \text{ N}\end{aligned}$$

**Ans.**

The magnitude of the component of  $\mathbf{F}$  perpendicular to segment  $BC$  of the pipe assembly can be determined from

$$(F_{BC})_{\text{per}} = \sqrt{F^2 - (F_{BC})_{\text{para}}^2} = \sqrt{400^2 - 244.95^2} = 316 \text{ N} \quad \text{Ans.}$$



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\*2-124. Cable  $OA$  is used to support column  $OB$ . Determine the angle  $\theta$  it makes with beam  $OC$ .

**Unit Vector :**

$$\mathbf{u}_{OC} = \mathbf{i}$$

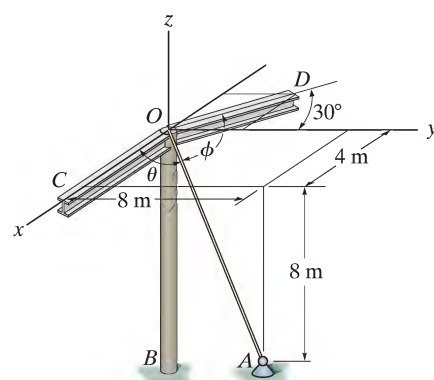
$$\begin{aligned}\mathbf{u}_{OA} &= \frac{(4-0)\mathbf{i} + (8-0)\mathbf{j} + (-8-0)\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}} \\ &= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\end{aligned}$$

**The Angle Between Two Vectors  $\theta$  :**

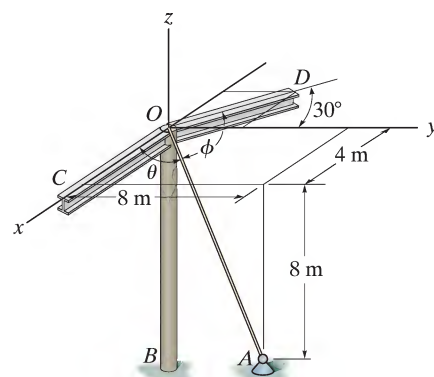
$$\mathbf{u}_{OC} \cdot \mathbf{u}_{OA} = (\mathbf{i}) \cdot \left( \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right) = 1\left(\frac{1}{3}\right) + (0)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right) = \frac{1}{3}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{OC} \cdot \mathbf{u}_{OA}) = \cos^{-1}\frac{1}{3} = 70.5^\circ \quad \text{Ans}$$



•2-125. Cable  $OA$  is used to support column  $OB$ . Determine the angle  $\phi$  it makes with beam  $OD$ .



**Unit Vector :**

$$\mathbf{u}_{OD} = -\sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j} = -0.5\mathbf{i} + 0.8660\mathbf{j}$$

$$\begin{aligned}\mathbf{u}_{OA} &= \frac{(4-0)\mathbf{i} + (8-0)\mathbf{j} + (-8-0)\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}} \\ &= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\end{aligned}$$

**The Angles Between Two Vectors  $\phi$  :**

$$\begin{aligned}\mathbf{u}_{OD} \cdot \mathbf{u}_{OA} &= (-0.5\mathbf{i} + 0.8660\mathbf{j}) \cdot \left( \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right) \\ &= (-0.5)\left(\frac{1}{3}\right) + (0.8660)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right) \\ &= 0.4107\end{aligned}$$

Then,

$$\phi = \cos^{-1}(\mathbf{u}_{OD} \cdot \mathbf{u}_{OA}) = \cos^{-1}0.4107 = 65.8^\circ \quad \text{Ans}$$

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**2–126.** The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of  $\mathbf{F}_1$  along the line of action of  $\mathbf{F}_2$ .

**Force Vector :**

$$\mathbf{u}_{F_1} = \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k} \\ = 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1 \mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \text{ N} \\ = \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\} \text{ N}$$

**Unit Vector :** The unit vector along the line of action of  $\mathbf{F}_2$  is

$$\mathbf{u}_{F_2} = \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k} \\ = 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

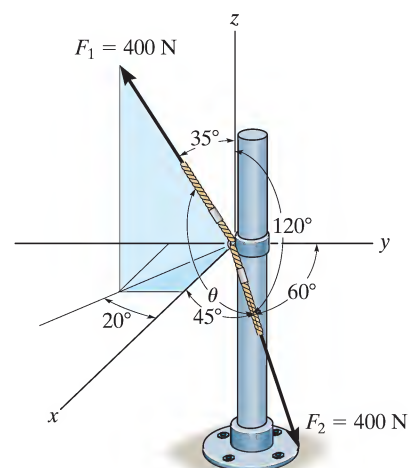
**Projected Component of  $\mathbf{F}_1$  Along Line of Action of  $\mathbf{F}_2$  :**

$$(\mathbf{F}_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}) \\ = (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5) \\ = -50.6 \text{ N}$$

Negative sign indicates that the force component  $(\mathbf{F}_1)_{F_2}$  acts in the opposite sense of direction to that of  $\mathbf{u}_{F_2}$ .

thus the magnitude is  $(\mathbf{F}_1)_{F_2} = 50.6 \text{ N}$

**Ans**



**2–127.** Determine the angle  $\theta$  between the two cables attached to the post.

**Unit Vector :**

$$\mathbf{u}_{F_1} = \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k} \\ = 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

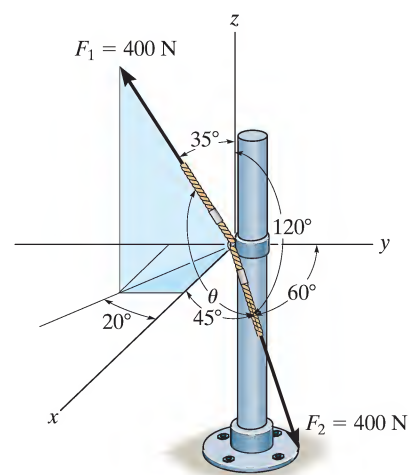
$$\mathbf{u}_{F_2} = \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k} \\ = 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

**The Angle Between Two Vectors  $\theta$  :** The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}) \\ = 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5) \\ = -0.1265$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}) = \cos^{-1}(-0.1265) = 97.3^\circ \quad \text{Ans}$$



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\*2-128. A force of  $F = 80$  N is applied to the handle of the wrench. Determine the angle  $\theta$  between the tail of the force and the handle  $AB$ .

$$\mathbf{u}_F = -\cos 30^\circ \sin 45^\circ \mathbf{i} + \cos 30^\circ \cos 45^\circ \mathbf{j} + \sin 30^\circ \mathbf{k}$$

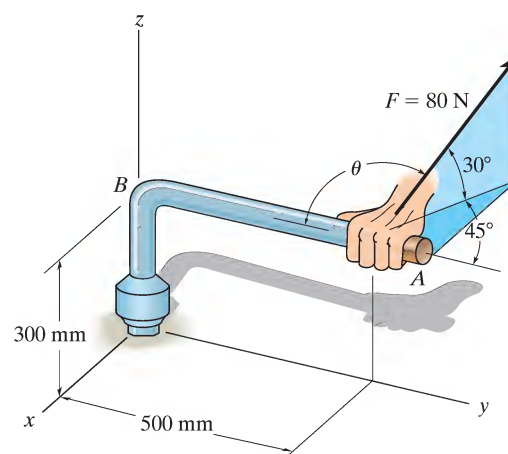
$$= -0.6124 \mathbf{i} + 0.6124 \mathbf{j} + 0.5 \mathbf{k}$$

$$\mathbf{u}_{AB} = -\mathbf{j}$$

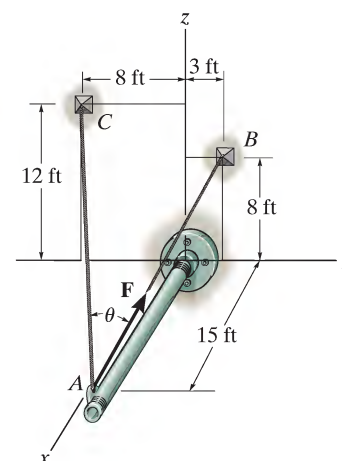
$$\cos \theta = \mathbf{u}_F \cdot \mathbf{u}_{AB} = (-0.6124 \mathbf{i} + 0.6124 \mathbf{j} + 0.5 \mathbf{k}) \cdot (-\mathbf{j})$$

$$= -0.6124$$

$$\theta = 128^\circ \quad \text{Ans}$$



•2-129. Determine the angle  $\theta$  between cables  $AB$  and  $AC$ .



**Position Vector :**

$$\begin{aligned} \mathbf{r}_{AB} &= \{(0-15)\mathbf{i} + (3-0)\mathbf{j} + (8-0)\mathbf{k}\} \text{ ft} \\ &= \{-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}\} \text{ ft} \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{AC} &= \{(0-15)\mathbf{i} + (-8-0)\mathbf{j} + (12-0)\mathbf{k}\} \text{ ft} \\ &= \{-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k}\} \text{ ft} \end{aligned}$$

The magnitudes of the position vectors are

$$r_{AB} = \sqrt{(-15)^2 + 3^2 + 8^2} = 17.263 \text{ ft}$$

$$r_{AC} = \sqrt{(-15)^2 + (-8)^2 + 12^2} = 20.809 \text{ ft}$$

**The Angle Between Two Vectors  $\theta$  :**

$$\begin{aligned} \mathbf{r}_{AB} \cdot \mathbf{r}_{AC} &= (-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) \cdot (-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k}) \\ &= (-15)(-15) + (3)(-8) + 8(12) \\ &= 297 \text{ ft}^2 \end{aligned}$$

Then,

$$\theta = \cos^{-1} \left( \frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{r_{AB} r_{AC}} \right) = \cos^{-1} \left[ \frac{297}{17.263(20.809)} \right] = 34.2^\circ \quad \text{Ans}$$

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**2-130.** If  $\mathbf{F}$  has a magnitude of 55 lb, determine the magnitude of its projected components acting along the  $x$  axis and along cable  $AC$ .

**Force Vector :**

$$\mathbf{u}_{AB} = \frac{(0-15)\mathbf{i} + (3-0)\mathbf{j} + (8-0)\mathbf{k}}{\sqrt{(0-15)^2 + (3-0)^2 + (8-0)^2}}$$

$$= -0.8689\mathbf{i} + 0.1738\mathbf{j} + 0.4634\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 55(-0.8689\mathbf{i} + 0.1738\mathbf{j} + 0.4634\mathbf{k}) \text{ lb}$$

$$= \{-47.79\mathbf{i} + 9.558\mathbf{j} + 25.489\mathbf{k}\} \text{ lb}$$

**Unit Vector :** The unit vector along negative  $x$  axis and  $AC$  are

$$\mathbf{u}_x = -\mathbf{i}$$

$$\mathbf{u}_{AC} = \frac{(0-15)\mathbf{i} + (-8-0)\mathbf{j} + (12-0)\mathbf{k}}{\sqrt{(0-15)^2 + (-8-0)^2 + (12-0)^2}}$$

$$= -0.7209\mathbf{i} - 0.3845\mathbf{j} + 0.5767\mathbf{k}$$

**Projected Component of  $\mathbf{F}$  :**

$$F_x = \mathbf{F} \cdot \mathbf{u}_x = (-47.79\mathbf{i} + 9.558\mathbf{j} + 25.489\mathbf{k}) \cdot (-\mathbf{i})$$

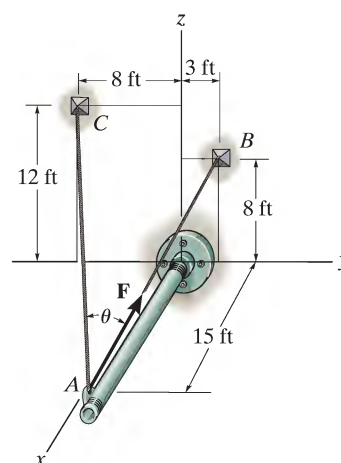
$$= (-47.79)(-1) + 9.558(0) + 25.489(0)$$

$$= 47.8 \text{ lb} \quad \text{Ans}$$

$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (-47.79\mathbf{i} + 9.558\mathbf{j} + 25.489\mathbf{k}) \cdot (-0.7209\mathbf{i} - 0.3845\mathbf{j} + 0.5767\mathbf{k})$$

$$= (-47.79)(-0.7209) + (9.558)(-0.3845) + (25.489)(0.5767)$$

$$= 45.5 \text{ lb} \quad \text{Ans}$$



**2-131.** Determine the magnitudes of the projected components of the force  $F = 300 \text{ N}$  acting along the  $x$  and  $y$  axes.

**Force Vector:** The force vector  $\mathbf{F}$  must be determined first. From Fig.  $a$ ,

$$\mathbf{F} = -300 \sin 30^\circ \sin 30^\circ \mathbf{i} + 300 \cos 30^\circ \mathbf{j} + 300 \sin 30^\circ \cos 30^\circ \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

**Vector Dot Product:** The magnitudes of the projected component of  $\mathbf{F}$  along the  $x$  and  $y$  axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{j}$$

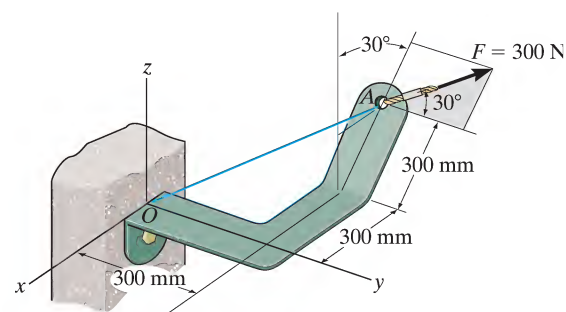
$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

The negative sign indicates that  $F_x$  is directed towards the negative  $x$  axis. Thus

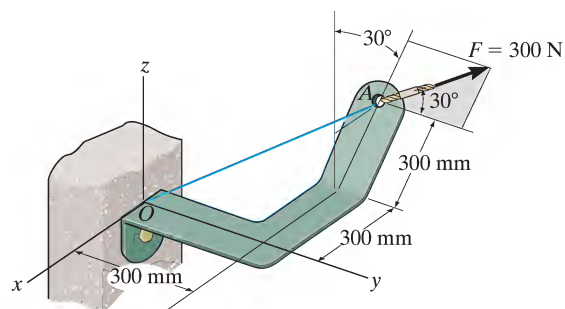
$$F_x = 75 \text{ N}, \quad F_y = 260 \text{ N}$$

**Ans.**



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**\*2-132.** Determine the magnitude of the projected component of the force  $F = 300$  N acting along line  $OA$ .



**Force and unit Vector:** The force vector  $\mathbf{F}$  and unit vector  $\mathbf{u}_{OA}$  must be determined first.

From Fig. (a)

$$\mathbf{F} = -300 \sin 30^\circ \sin 30^\circ \mathbf{i} + 300 \cos 30^\circ \mathbf{j} + 300 \sin 30^\circ \cos 30^\circ \mathbf{k}$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

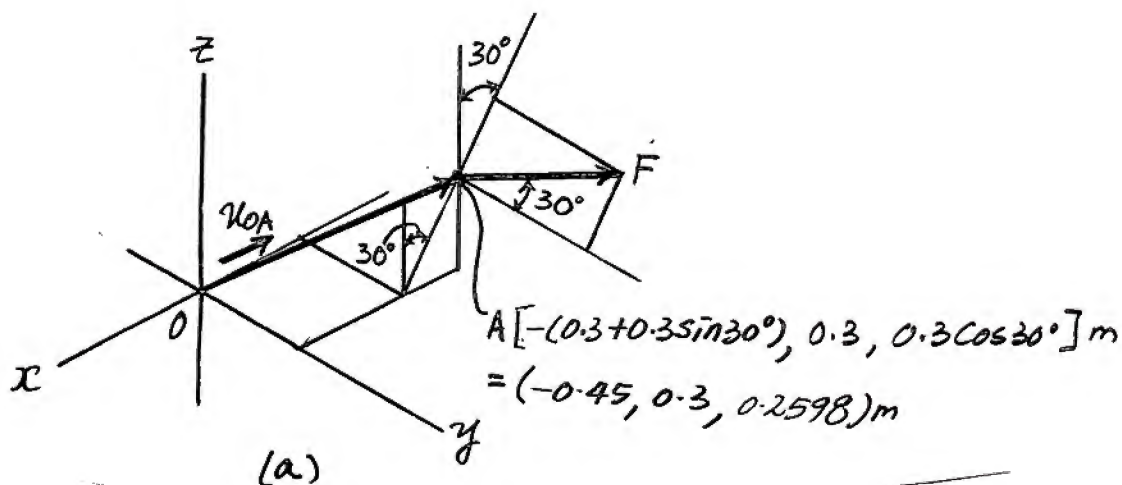
**Vector Dot Product:** The magnitude of the projected component of  $\mathbf{F}$  along line  $OA$  is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

$$= 242 \text{ N}$$

**Ans.**



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•2–133. Two cables exert forces on the pipe. Determine the magnitude of the projected component of  $F_1$  along the line of action of  $F_2$ .

**Force Vector :**

$$u_{F_1} = \cos 30^\circ \sin 30^\circ i + \cos 30^\circ \cos 30^\circ j - \sin 30^\circ k \\ = 0.4330i + 0.75j - 0.5k$$

$$F_1 = F_1 u_{F_1} = 30(0.4330i + 0.75j - 0.5k) \text{ lb} \\ = \{12.990i + 22.5j - 15.0k\} \text{ lb}$$

**Unit Vector :** One can obtain the angle  $\alpha = 135^\circ$  for  $F_2$  using Eq. 2–53.  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ , with  $\beta = 60^\circ$  and  $\gamma = 60^\circ$ . The unit vector along the line of action of  $F_2$  is

$$u_{F_2} = \cos 135^\circ i + \cos 60^\circ j + \cos 60^\circ k = -0.7071i + 0.5j + 0.5k$$

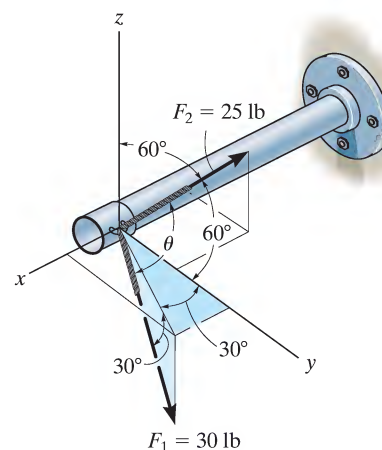
**Projected Component of  $F_1$  Along the Line of Action of  $F_2$  :**

$$(F_1)_{F_2} = F_1 \cdot u_{F_2} = (12.990i + 22.5j - 15.0k) \cdot (-0.7071i + 0.5j + 0.5k) \\ = (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5) \\ = -5.44 \text{ lb}$$

Negative sign indicates that the projected component  $(F_1)_{F_2}$  acts in the opposite sense of direction to that of  $u_{F_2}$ .

The magnitude is  $(F_1)_{F_2} = 5.44 \text{ lb}$ .

**Ans**



2–134. Determine the angle  $\theta$  between the two cables attached to the pipe.

**The Angles Between Two Vectors  $\theta$  :**

$$u_{F_1} \cdot u_{F_2} = (0.4330i + 0.75j - 0.5k) \cdot (-0.7071i + 0.5j + 0.5k) \\ = 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5) \\ = -0.1812$$

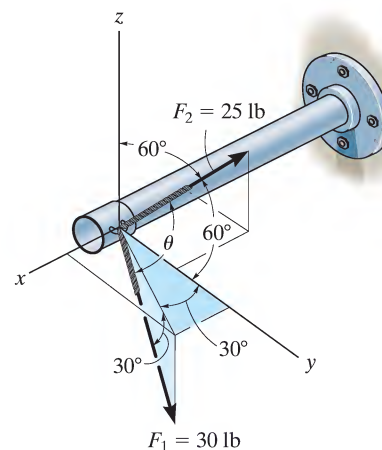
Then,

$$\theta = \cos^{-1}(u_{F_1} \cdot u_{F_2}) = \cos^{-1}(-0.1812) = 100^\circ \quad \text{Ans}$$

**Unit Vector :**

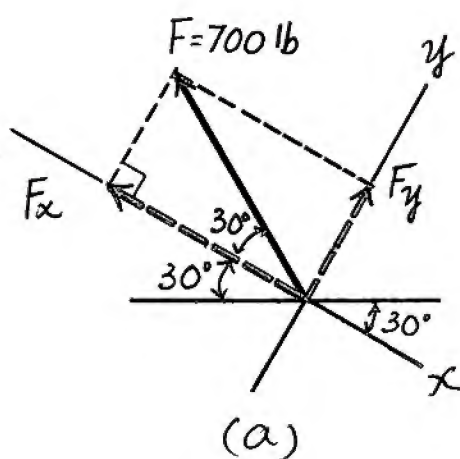
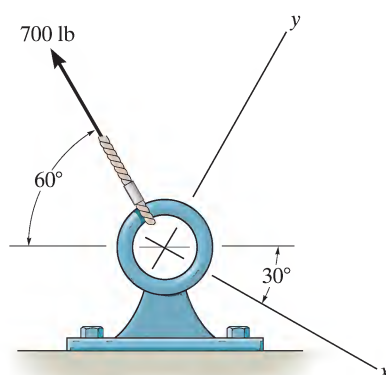
$$u_{F_1} = \cos 30^\circ \sin 30^\circ i + \cos 30^\circ \cos 30^\circ j - \sin 30^\circ k \\ = 0.4330i + 0.75j - 0.5k$$

$$u_{F_2} = \cos 135^\circ i + \cos 60^\circ j + \cos 60^\circ k \\ = -0.7071i + 0.5j + 0.5k$$



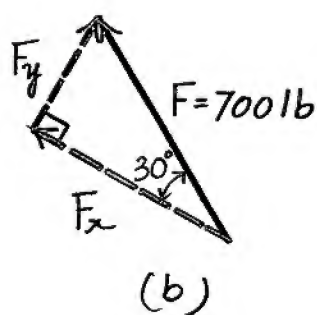
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**2-135.** Determine the  $x$  and  $y$  components of the 700-lb force.



$$F_x = -700 \cos 30^\circ = -606 \text{ lb} \quad \text{Ans}$$

$$F_y = 700 \sin 30^\circ = 350 \text{ lb} \quad \text{Ans}$$



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\*2-136. Determine the magnitude of the projected component of the 100-lb force acting along the axis  $BC$  of the pipe.

**Force Vector :**

$$\mathbf{u}_{CD} = \frac{(0-6)\mathbf{i} + (12-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (12-4)^2 + [0-(-2)]^2}}$$

$$= -0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{CD} = 100(-0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k})$$

$$= \{-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}\} \text{ lb}$$

**Unit Vector :** The unit vector along  $CB$  is

$$\mathbf{u}_{CB} = \frac{(0-6)\mathbf{i} + (0-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (0-4)^2 + [0-(-2)]^2}}$$

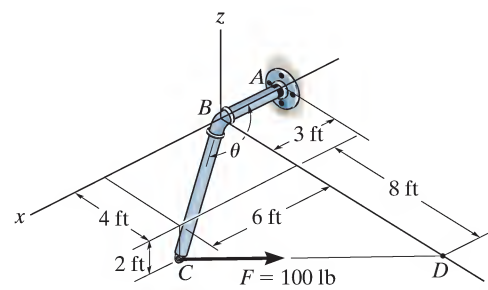
$$= -0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k}$$

**Projected Component of  $\mathbf{F}$  Along  $CB$  :**

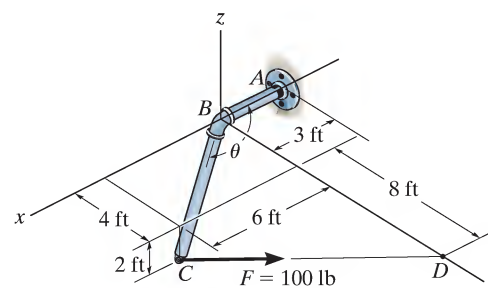
$$F_{CB} = \mathbf{F} \cdot \mathbf{u}_{CB} = (-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}) \cdot (-0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k})$$

$$= (-58.835)(-0.8018) + (78.446)(-0.5345) + (19.612)(0.2673)$$

$$= 10.5 \text{ lb} \quad \text{Ans}$$



•2-137. Determine the angle  $\theta$  between pipe segments  $BA$  and  $BC$ .



**Position Vector :**

$$\mathbf{r}_{BA} = \{-3\mathbf{i}\} \text{ ft}$$

$$\mathbf{r}_{BC} = \{(6-0)\mathbf{i} + (4-0)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft}$$

$$= \{6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

The magnitudes of the position vectors are

$$r_{BA} = 3.00 \text{ ft} \quad r_{BC} = \sqrt{6^2 + 4^2 + (-2)^2} = 7.483 \text{ ft}$$

**The Angle Between Two Vectors  $\theta$  :**

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i}) \cdot (6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$= (-3)(6) + (0)(4) + 0(-2)$$

$$= -18.0 \text{ ft}^2$$

Then,

$$\theta = \cos^{-1} \left( \frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}} \right) = \cos^{-1} \left[ \frac{-18.0}{3.00(7.483)} \right] = 143^\circ \quad \text{Ans}$$

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2-138. Determine the magnitude and direction of the resultant  $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$  of the three forces by first finding the resultant  $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$  and then forming  $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$ . Specify its direction measured counter-clockwise from the positive  $x$  axis.

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50) \cos 105^\circ} = 104.7 \text{ N}$$

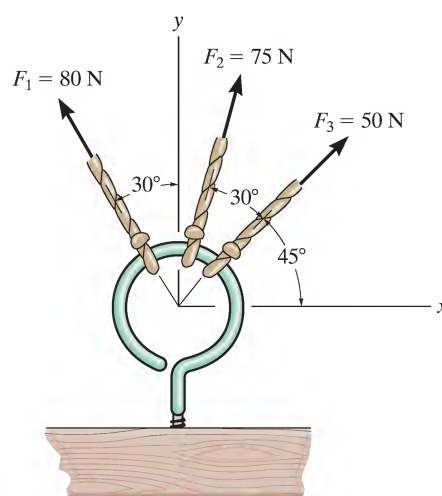
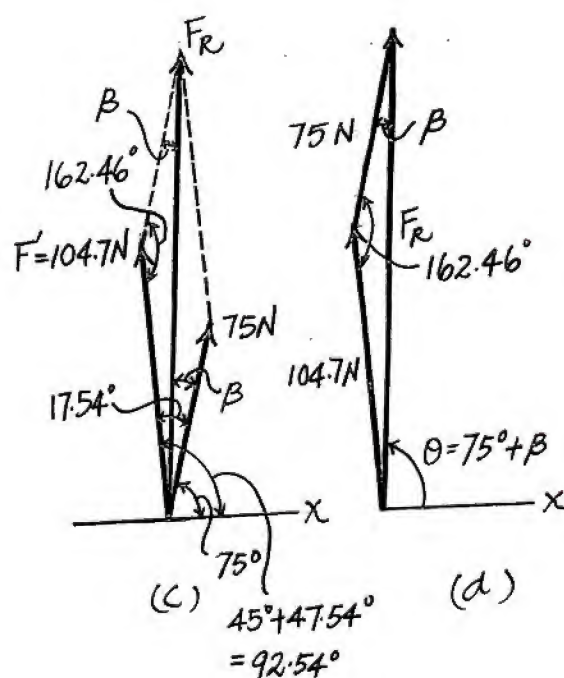
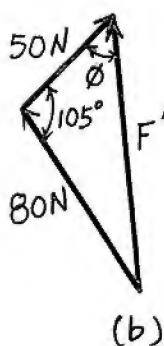
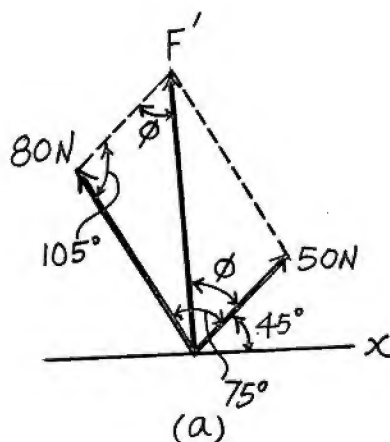
$$\frac{\sin \phi}{80} = \frac{\sin 105^\circ}{104.7}; \quad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75) \cos 162.46^\circ}$$

$$F_R = 177.7 = 178 \text{ N} \quad \text{Ans}$$

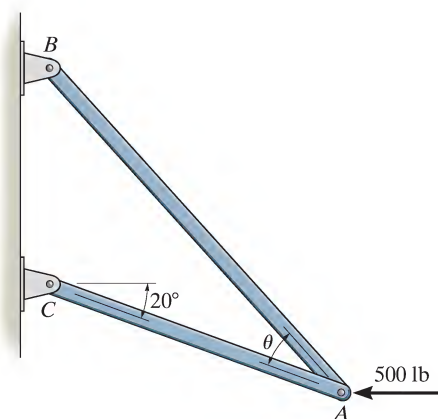
$$\frac{\sin \beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \quad \beta = 10.23^\circ$$

$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ \quad \text{Ans}$$



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**2-139.** Determine the design angle  $\theta$  ( $\theta < 90^\circ$ ) between the two struts so that the 500-lb horizontal force has a component of 600 lb directed from  $A$  toward  $C$ . What is the component of force acting along member  $BA$ ?



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{500^2 + 600^2 - 2(500)(600)\cos 20^\circ}$$

$$= 214.91 \text{ lb} \approx 215 \text{ lb}$$

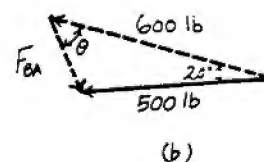
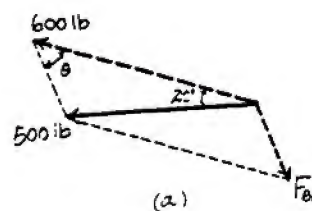
**Ans.**

Applying the law of sines to Fig. *b* and using this result yields

$$\frac{\sin \theta}{500} = \frac{\sin 20^\circ}{214.91}$$

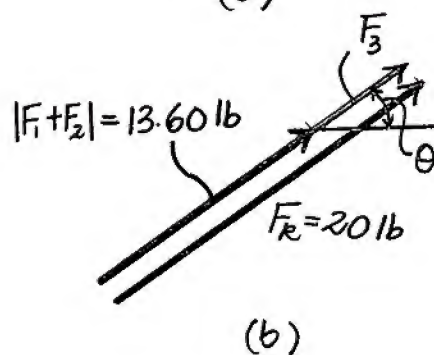
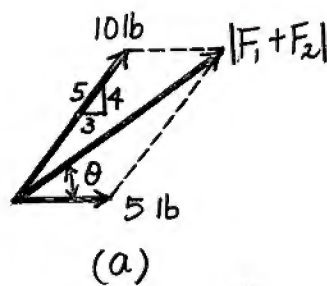
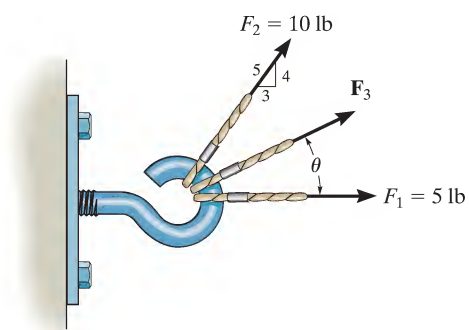
$$\theta = 52.7^\circ$$

**Ans.**



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**\*2-140.** Determine the magnitude and direction of the *smallest* force  $\mathbf{F}_3$  so that the resultant force of all three forces has a magnitude of 20 lb.



$\mathbf{F}_3$  is minimum :

$$F_3 = 20 - |\mathbf{F}_1 + \mathbf{F}_2|$$

$$\mathbf{F}_1 + \mathbf{F}_2 = \left(5 + 10\left(\frac{3}{5}\right)\right)\mathbf{i} + \left(10\left(\frac{4}{5}\right)\right)\mathbf{j} = 11\mathbf{i} + 8\mathbf{j}$$

$$|\mathbf{F}_1 + \mathbf{F}_2| = \sqrt{11^2 + 8^2} = 13.601 \text{ lb}$$

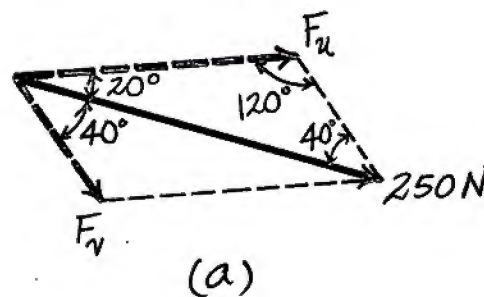
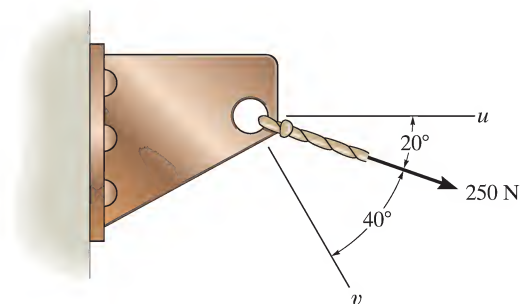
$$\theta = \tan^{-1}\left(\frac{8}{11}\right) = 36.0^\circ \quad \text{Ans}$$

Thus

$$(F_3)_{\min} = 20 - 13.601 = 6.40 \text{ lb} \quad \text{Ans}$$

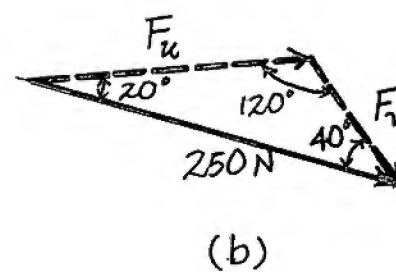
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•2-141. Resolve the 250-N force into components acting along the  $u$  and  $v$  axes and determine the magnitudes of these components.



$$\frac{250}{\sin 120^\circ} = \frac{F_u}{\sin 40^\circ}; \quad F_u = 186 \text{ N} \quad \text{Ans}$$

$$\frac{250}{\sin 120^\circ} = \frac{F_v}{\sin 20^\circ}; \quad F_v = 98.7 \text{ N} \quad \text{Ans}$$



2-142. Cable  $AB$  exerts a force of 80 N on the end of the 3-m-long boom  $OA$ . Determine the magnitude of the projection of this force along the boom.

Vector Analysis :

$$\mathbf{F} = 80 \left( \frac{\mathbf{r}_{AB}}{r_{AB}} \right) = 80 \left( -\frac{3 \cos 60^\circ}{5} \mathbf{i} - \frac{3 \sin 60^\circ}{5} \mathbf{j} + \frac{4}{5} \mathbf{k} \right)$$

$$= -24 \mathbf{i} - 41.57 \mathbf{j} + 64 \mathbf{k}$$

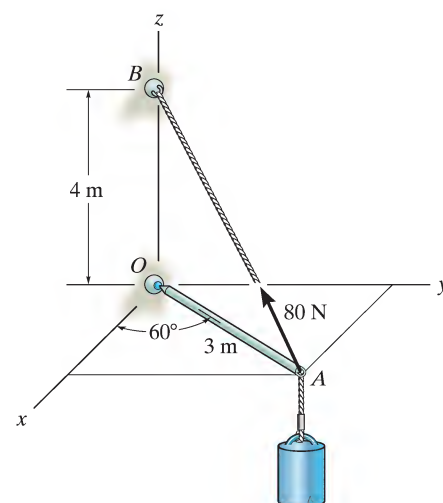
$$\mathbf{u}_{AO} = -\cos 60^\circ \mathbf{i} - \sin 60^\circ \mathbf{j} = -0.5 \mathbf{i} - 0.866 \mathbf{j}$$

$$\text{Proj}_{\mathbf{F}} \mathbf{F} = \mathbf{F} \cdot \mathbf{u}_{AO} = (-24)(-0.5) + (-41.57)(-0.866) + (64)(0) = 48.0 \text{ N} \quad \text{Ans}$$

Scalar Analysis :

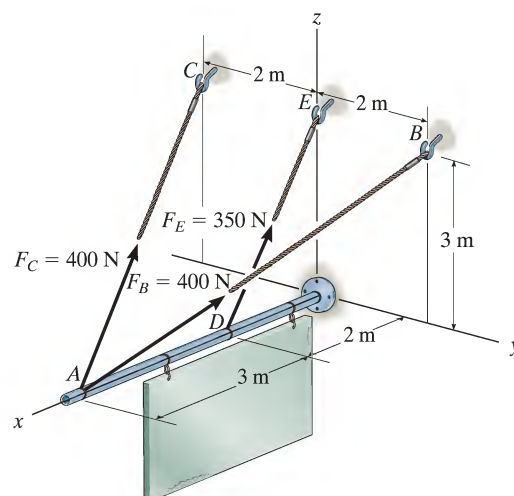
$$\text{Angle } OAB = \tan^{-1} \left( \frac{4}{3} \right) = 53.13^\circ$$

$$\text{Proj } \mathbf{F} = 80 \cos 53.13^\circ = 48.0 \text{ N} \quad \text{Ans}$$



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**2–143.** The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.



**Unit Vector:**

$$\mathbf{r}_{AB} = \{(0-5)\mathbf{i} + (2-0)\mathbf{j} + (3-0)\mathbf{k}\} \text{ m} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{(-5)^2 + 2^2 + 3^2} = 6.164 \text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{6.164} = -0.8111\mathbf{i} + 0.3244\mathbf{j} + 0.4867\mathbf{k}$$

$$\mathbf{r}_{AC} = \{(0-5)\mathbf{i} + (-2-0)\mathbf{j} + (3-0)\mathbf{k}\} \text{ m} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(-5)^2 + (-2)^2 + 3^2} = 6.164 \text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{6.164} = -0.8111\mathbf{i} - 0.3244\mathbf{j} + 0.4867\mathbf{k}$$

$$\mathbf{r}_{DE} = \{(0-2)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k}\} \text{ m} = \{-2\mathbf{i} + 3\mathbf{k}\} \text{ m}$$

$$r_{DE} = \sqrt{(-2)^2 + 3^2} = 3.605 \text{ m}$$

$$\mathbf{u}_{DE} = \frac{\mathbf{r}_{DE}}{r_{DE}} = \frac{-2\mathbf{i} + 3\mathbf{k}}{3.605} = -0.5547\mathbf{i} + 0.8321\mathbf{k}$$

**Force Vector:**

$$\begin{aligned} \mathbf{F}_B &= F_B \mathbf{u}_{AB} = 400\{-0.8111\mathbf{i} + 0.3244\mathbf{j} + 0.4867\mathbf{k}\} \text{ N} \\ &= \{-324.44\mathbf{i} + 129.78\mathbf{j} + 194.67\mathbf{k}\} \text{ N} \\ &= \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\} \text{ N} \end{aligned}$$

**Ans**

$$\begin{aligned} \mathbf{F}_C &= F_C \mathbf{u}_{AC} = 400\{-0.8111\mathbf{i} - 0.3244\mathbf{j} + 0.4867\mathbf{k}\} \text{ N} \\ &= \{-324.44\mathbf{i} - 129.78\mathbf{j} + 194.67\mathbf{k}\} \text{ N} \\ &= \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\} \text{ N} \end{aligned}$$

**Ans**

$$\begin{aligned} \mathbf{F}_E &= F_E \mathbf{u}_{DE} = 350\{-0.5547\mathbf{i} + 0.8321\mathbf{k}\} \text{ N} \\ &= \{-194.15\mathbf{i} + 291.22\mathbf{k}\} \text{ N} \\ &= \{-194\mathbf{i} + 291\mathbf{k}\} \text{ N} \end{aligned}$$

**Ans**